STATISTICAL RESEARCH OF THE NANOSATELLITE RELATIVE MOTION AFTER SEPARATION FROM THE ROCKET CARRIER UPPER STAGE

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This paper is dedicated to the problem of upper stages use for a nanosatellite low-cost orbiting as a piggy back launch (for example, the rocket "Soyuz"). Upper stages after separation of the primary payload make uncontrollable flight and become space debris. Reliable data on angular velocities the stage obtained are not available, because it was not expected their use. Statistical modeling of the primary payload separation process allowed to calculate estimates of the angular velocity projections, which obtains the upper stage (for the upper stage of the rocket carrier "Soyuz" their range is +/– 2.5 deg/s). These data allowed to formulate a new task to provide the safe separation of the secondary payload (nanosatellite) from the upper stage for which the orientation of the longitudinal axis is described by a probabilistic model. In this problem, when the motion at low orbits due to the large differences between ballistic coefficients of upper stage and nanosatellite, the atmospheric drag can occur dangerous for two flight turns after the separation. Preliminary analysis confirmed the relevance of this problem and need choose the initial conditions of separation (time delay and the separation velocity of the nanosatellite) for guaranteeing of safe motion both nanosatellite and upper stage. There was obtained the analytical equations for the distribution laws of the nanosatellite separation velocity components in the random orientation of the upper stage longitudinal axis conditions. These equations are allowed to estimate the probability of the nanosatellite position in a dangerous area around the upper stage. The results obtained analytically are coincided with the results of the numerical simulation. There were made the estimations of the probability of possible dangerous approaches occurrence for Soyuz orbital stage (insertion orbit – heights in perigee 190 km, in apogee 240 km) and formed the recommendations for dangerous approaches prevention. The acceptable parameters of the nanosatellite separation with the random nature of the upper stage orientation were obtained. The separation velocity should be from 1 m/s till 1.5 m/s, and the time delay of the nanosatellite separation after the primary payload separation should be from 5 s till 20 s. The method of choice of separation parameters from non-oriented platform can use for piggy back launching not only for Soyuz upper stage but for other rockets. The reported study was partially supported by Russian Fund for Basic Research, research project No. 13-08-97015- r_Volga_region_a.

I. INTRODUCTION

Nowadays a lot of innovative companies and universities work on designing of nanosatellites (NS). Such satellites are so popular because their creation doesn't require investing a lot of finances. It is possible to test different sensors and elements of onboard systems in space conditions with NS. Besides, participation of young scientists in designing of NS opens opportunity to teach students space technology. But orbiting and flight tests are a big problem which solves with piggy back launch. This kind of launch is less expensive and it requires only a separation device. As usual every carrier-rocket has additional space to place NS. In paper [1] it was suggested to use the transfer compartment of the carrier-rocket "Soyuz" upper stage for these purposes.

II. UPPER STAGE PRECESSION

The motion of NS relatively to the orbital (upper) stage (OS) separated from as a piggy back payload is investigated. The nanosatellite is separated in a random direction, which is caused by uncontrolled flight of the OS around the center of its mass. According to the known distribution laws of the OS angular motion parameters after the primary payload separation there was obtained the analytical expressions for the distribution laws of the initial velocity projections NS separation in the orbital reference system. Nanosatellite is deployed from the transfer compartment of the carrier-rocket orbital stage with a certain time delay after the primary payload separation. It is considered the motion of the upper stage around mass center, neglecting the influence of external forces. It is assumed that the kinetic energy of the upper stage rotation is much larger than work of external forces. The orbital stage obtains rotational motion after the separation of the primary payload. This motion is a regular precession – the longitudinal axis of the stage passing through the mass center describes a circular cone relatively to the fixed direction of the kinetic momentum vector K (angle of a cone α_k) (fig. I). The motion of the symmetry axis around the kinetic momentum vector K

has a constant angular velocity of precession $\dot{\psi}$. At the same time stage rotates with a constant angular velocity $\dot{\varphi}$ of rotation around the symmetry axis [2].

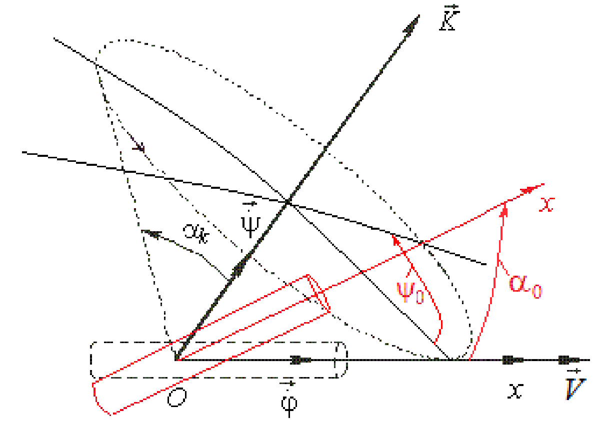

Fig. I: Initial conditions of the OS attitude motion

It is assumed that longitudinal axis coincides with the velocity of its mass center when the primary payload separates. After a certain time (time delay Δt) the NS is separated, the angle of attack α_0 (the angle between the longitudinal axis of the OS and the velocity vector) at the time of NS separation can be defined by the formula

$$\alpha_0 = \arccos(\cos^2(\alpha_k) + \sin^2(\alpha_k)\cos(\psi_0)), \qquad (1)$$

where $\cos(\alpha_k)=K_x/K$; $\dot{\psi}=K/J_n$; $\psi_0=\dot{\psi}\cdot\Delta t$ – precession angle in the NS separation moment; $K=\sqrt{K_x^2+K_n^2}$ – module of the OS kinetic moment; $K_x=I_x\omega_x$, $K_n=I_n\omega_n$ – longitudinal and transverse projections of the kinetic momentum; I_x – longitudinal inertia moment of the OS, $I_y=I_z=I_n$ – transverse inertia moment of the OS; ω_{x0}, $\omega_{n0}=\sqrt{\omega_{y0}^2+\omega_{z0}^2}$ – longitudinal and transverse projections of the OS angular velocity has normal distribution [3].

In paper [4] was obtained expression for probability density of angle α_0, which has Rayleigh distribution:

$$f(\alpha_0)=\frac{\alpha_0}{\sigma^2\Delta t^2}\exp\left(\frac{-\alpha_0^2}{2\sigma^2\Delta t^2}\right), \qquad (2)$$

where $\sigma^2=\sigma_{\omega_y}^2=\sigma_{\omega_z}^2$ – dispersions of angular velocity transverse projections which have a normal distributions.

The projection of the NS separation velocity module ΔV on the X-axis of the orbital reference system, which coincides with the velocity vector of the OS, is defined by the following equation:

$$V_x = F(\alpha_0) = \Delta V\cos\alpha_0 = \Delta V\cos^2(\alpha_k) + \Delta V\sin^2(\alpha_k)\cos(\psi_0)).$$

Using (1), it is obtained inverse function

$$\alpha_0 = F^{-1}(V_x) = \arccos(V_x/\Delta V).$$

Since the angle α_0 has a known law of distribution (2), it is possible obtain the distribution law for V_x from the formula

$$f(V_x)=f_{\alpha_0}\left(F^{-1}(V_x)\right)\left|\frac{dF^{-1}(V_x)}{dV_x}\right|,$$

where $f_{\alpha_0}\left(F^{-1}(V_x)\right)$ – the probability density of the inverse function $F^{-1}(V_x)$;

$$\left|\frac{dF^{-1}(V_x)}{dV_x}\right|=\frac{-1}{\Delta V\sqrt{1-V_x^2/\Delta V^2}}.$$

Thus, the distribution law for V_x is presented in view:

$$f(V_x)=\frac{\alpha_0}{\sigma^2\Delta t^2\Delta V\sqrt{1-\frac{V_x^2}{\Delta V^2}}}\exp\left(\frac{-\alpha_0^2}{2\sigma^2\Delta t^2}\right).$$

Similarly it was obtained a probability density distribution for the transverse projection V_n:

$$V_n = \Delta V\sin\alpha_0 \approx \Delta V\alpha_0$$

$$f(V_n)=\frac{\alpha_0}{\sigma^2\Delta t^2\Delta V\sqrt{1-\frac{V_n^2}{\Delta V^2}}}\exp\left(\frac{-\alpha_0^2}{2\sigma^2\Delta t^2}\right).$$

As can be seen from the result, the transverse projection of the NS separation velocity will also be distributed by the Rayleigh law, since the angle α_0 distributed by the Rayleigh law. Consequently, the projections of V_y and V_z are normally distributed:

$$f(V_y)=\frac{\exp\left(\frac{-V_y{}^2}{2\sigma^2\Delta t^2}\right)}{2\cdot\sigma\cdot\Delta t\cdot\Delta V\sqrt{2\pi}},$$

$$f(V_z)=\frac{\exp\left(\frac{-V_z{}^2}{2\sigma^2\Delta t^2}\right)}{2\cdot\sigma\cdot\Delta t\cdot\Delta V\sqrt{2\pi}}.$$

It was made a numerical simulation of the separation velocity projections distribution with statistical tests method to confirm the correctness of the analytical expressions. Fig. II - IV show a histogram of the empirical distribution function model of the NS separation velocity projections and obtained analytical models graphics. Histograms are obtained by processing the random sample (10000 numerical experiments) formed under the following conditions: NS separation velocity ΔV = 1 m/s, NS separation time delay Δt = 20 s.

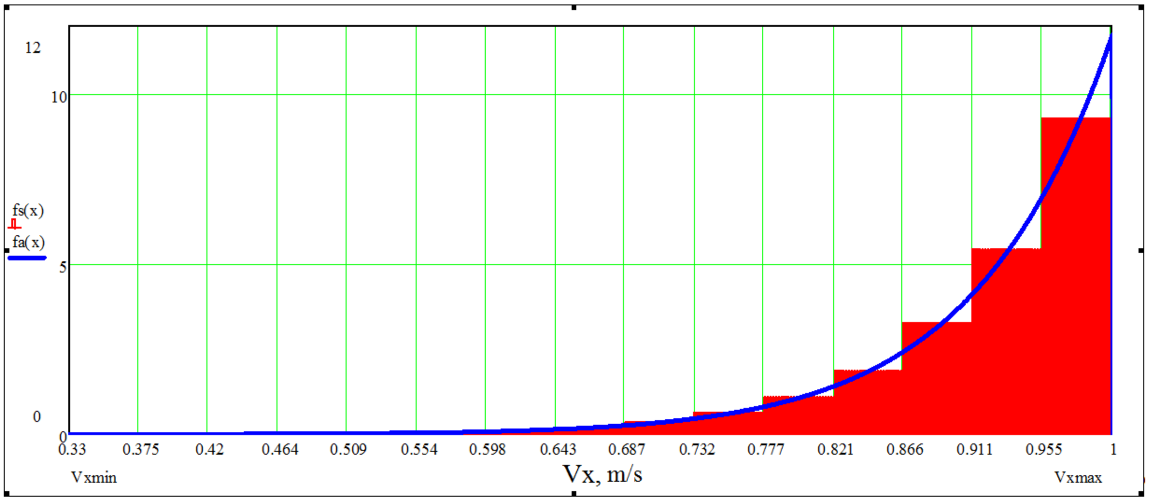

Fig. II: Projection V_x of the separation velocity distribution

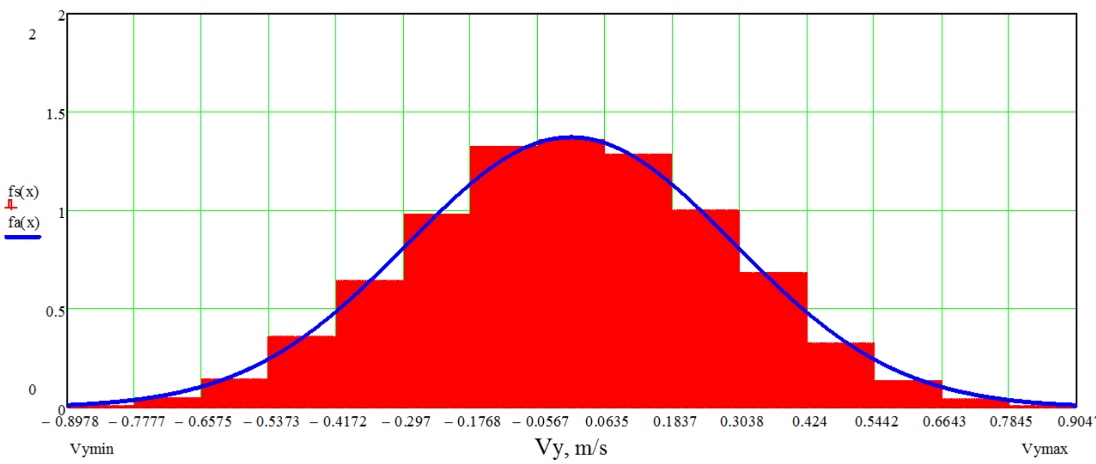

Fig. III: Projection V_y of the separation velocity distribution

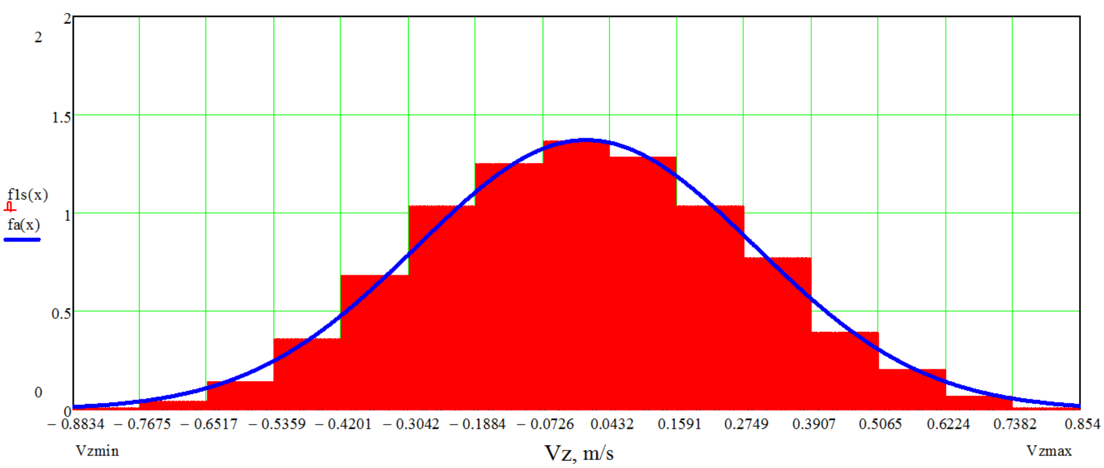

Fig. IV: Projection V_z of the separation velocity distribution

The obtained analytical model can be used to create the procedure for the selection of NS separation conditions to provide the safe orbital motion. Reliability of the obtained model was confirmed by numerical simulation.

III. STATISTICAL ESTIMATION OF THE DANGEROUS APPROACH PROBABILITY

After the separation of the main payload (MP) OS obtains the angular velocity which has a random magnitude and direction. Thus the NS separation will take place in a random direction. This direction will depend on the angular velocity of the OS and the NS separation time delay after the separation of the MP. Because of the considerable differences between the the OS and NS ballistic coefficient, aerodynamic drag for these objects will be different, which makes their relative motion special and under certain conditions can lead to dangerous approach. Thus, it is investigated the probability of such NS separation conditions, which could lead to the possibility of its dangerous approaches with the OS and/or MP, and it will be assessed the probability of hitting the NS in a dangerous area around the OS and MP (fig. V).

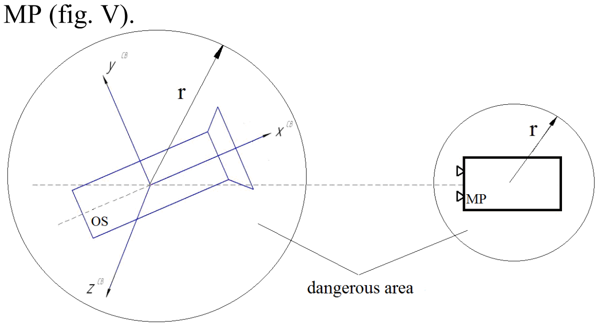

Fig. V: Dangerous area

For a stochastic analysis of the possibility of dangerous approach between nanosatellite and orbital stage or the main payload it was chosen method of statistical tests. Since the values of angular velocity projections are random, but their range is known, it is necessary to generate a sample of these values in a determined range and make multiple modeling of OS motion around its mass center after separating the main MP but before the separation of the NS. Also it is necessary to model the NS motion after its separation regarding to the OS.

It was assumed the hypothesis of a normal distribution of OS angular velocity projections ω_x, ω_y and ω_z. There were formed random sampling of input factors and output random variables of 10000 values, which corresponds to an error in the probability estimates characteristics calculation that do not exceed 1%.

After defining the angular velocity projections which OS has after the MP separation it is necessary to determine the NS separation velocity projections in the orbital reference system.

It is necessary to determine the initial conditions of NS separation (V_{x0}, V_{y0}, V_{z0}) and to simulate the relative motion. Mathematical model of relative motion for OS which has elliptic orbit (190 km × 240 km) [5]

$$\begin{cases} \ddot{x} + 2\dot{\theta}\,\dot{y} + \ddot{\theta}\,y - \dot{\theta}^2 x - \dfrac{\mu}{R_0^3}x = P_x \\ \ddot{y} - 2\dot{\theta}\,\dot{x} - \ddot{\theta}\,x - \dot{\theta}^2 y - \dfrac{\mu}{R_0^2} + \dfrac{\mu}{R^3}\left(y + R\right) = 0 \\ \ddot{z} + \dfrac{\mu}{R_0^3}z = 0 \end{cases} \quad (3)$$

where $\dot{\theta} = \dfrac{\sqrt{\mu p}}{R^2}$, $\ddot{\theta} = -2e\sqrt{\dfrac{\mu}{p^3}}\dot{\theta}\sin\theta$ – expression of the first and second derivatives of the angle of the true anomaly θ, respectively; $R_0 = [x^2 + (R+y)^2 + z^2]^{1/2}$ – radius-vector of the NS; $R = p/(1+e\cdot\cos\theta)$ – radius-vector of the OS; p – focal parameter of the orbit; e – eccentricity; $P_x = a^{OS} - a^{NS}$ – projection of the aerodynamic acceleration; $a^{NS} = S_b^{NS}\rho V^2$ – aerodynamic acceleration of the nanosatellite; $a^{OS} = S_b^{OS}\rho V^2$ – aerodynamic acceleration of the orbital stage; $\Delta Q = S_b^{OS} - S_b^{NS}$ – ballistic coefficients difference between OS and NS; S_b^{OS} – ballistic coefficient of the OS; S_b^{NS} – ballistic coefficient of the NS; ρ – atmospheric density; V – the incoming flow velocity.

Time interval for simulation is 2 orbits of the carrier-rocket "Soyuz" OS (10640 s).

According to the results of modeling to determine the distance of the closest approach of the NS with OS and with the MP for a chosen time period. Then it is checking the condition of the NS reaching a dangerous area around the OS or MP. It is forming the statistical estimations. In particular, it is determining the rate of the NS reaching a dangerous area around the OS or MP from the formula:

$$p = \frac{N_0}{N},$$

where N – number of simulations; N_0 – number of reaching a dangerous area.

IV. STATISTICAL SIMULATION OF THE NANOSATELLITE RELATIVE MOTION

The analysis was performed for the initial data of to the carrier-rocket "Soyuz" orbital stage: orbit 190 × 240 km, ballistic coefficient of the OS S_b^{OS} =0,002..0,007 m^2/kg, ballistic coefficient of the NS S_b^{NS} = 0,01 m^2/kg, ballistic coefficient of the MP S_b^{MP} =1,255·10^{-3} m^2/kg.

As a result of the research was obtained the dependence of the probability of the NS reaching the dangerous area around the OS (fig. V) on the radius of the area under different separation parameters.

Estimations of the probability of the NS reaching dangerous area around the OS (50, 100 and 200 meters), depending on the NS separation delay at various separation velocities are shown in fig. VI – VIII.

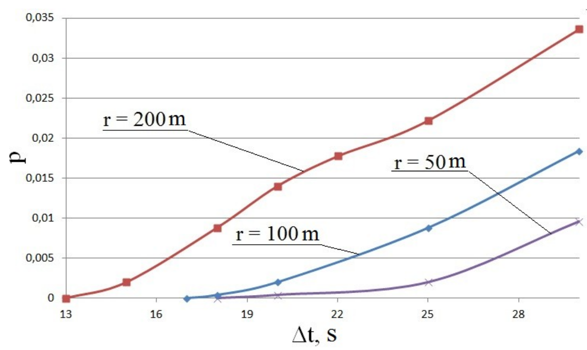

Fig. VI: Probability of the NS reaching dangerous area around the OS at separation velocity 0,5 m/s

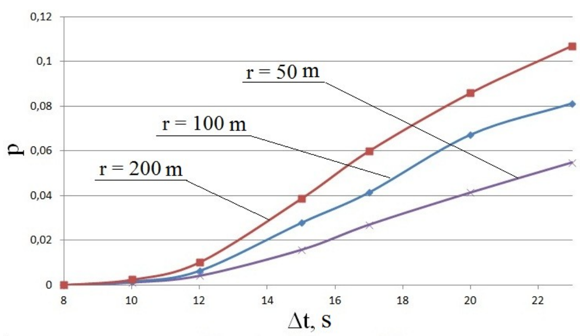

Fig. VII: Probability of the NS reaching dangerous area around the OS at separation velocity 1 m/s

For studying the probability of the NS reaching the dangerous area around MP (fig. V) it was made a simulation of relative motion using mathematical model (3) for the NS and MP. Time interval for MP motion simulation is 1 orbit (5320 s) because nowadays MP have boosters which take away on higher orbit after 35 minutes (2100 s). It makes the approaching between NS and MP impossible.

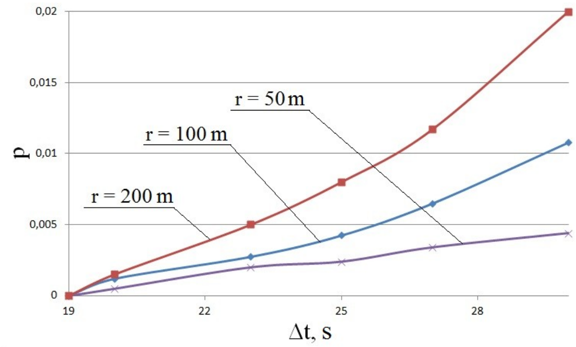

Fig. VIII: Probability of the NS reaching dangerous area around the OS at separation velocity 1,5 m/s

Estimations of the probability of the NS reaching dangerous area around the MP, depending on the NS separation velocity at various separation delays are shown in fig. IX and X. It was considered the dangerous area with radius 25 meters.

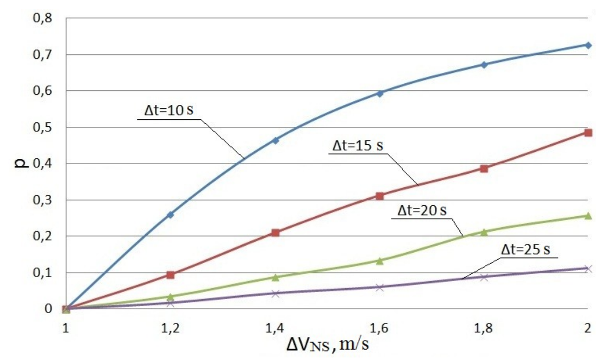

Fig. IX: Probability of the NS reaching dangerous area around the MP at MP separation velocity 1 m/s

On fig. IX and X we can see that probability of the NS reaching the dangerous area around MP decreases while separation time delay increases. Also we can see that there is a possibility of collision between NS and MP when the NS separation velocity is higher than the MP separation velocity.

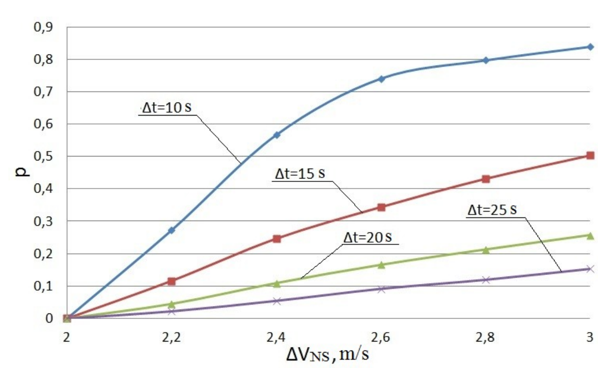

Fig. X: Probability of the NS reaching dangerous area around the MP at MP separation velocity 2 m/s

The formulation of requirements to a NS separation delay and velocity from the condition of noncollision between NS, OS and MP was made using mathematical model of relative motion (3). As it was mentioned earlier, the NS separation velocity should be lower than the MP separation velocity to avoid their collision.

There is the area of the NS separation parameters (separation velocity and time delay) on fig. XI and XII. Separation parameters in this area avoid any collisions between NS and OS with probability 0.997 and 0.99.

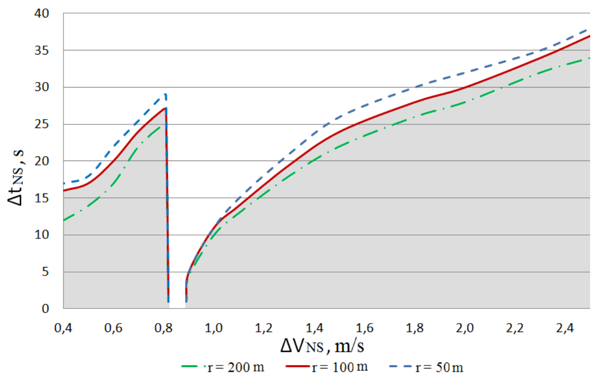

Fig. XI: The area of NS separation parameters which avoids any collisions between NS and OS with probability 0.997

Thus, choosing the NS separation parameters from fig. XI and XII (velocity and time delay), it is possible to provide safe NS motion with certain probability.

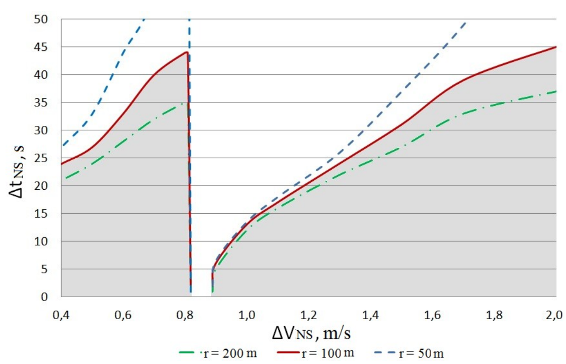

Fig. XII: The area of NS separation parameters which avoids any collisions between NS and OS with probability 0.99

V. CONCLUSION

It was made a statistical simulation of relative motion and it was formulated the requirements to the NS separation velocity and time delay from condition of noncollision NS with OS and MP. It was obtained the areas of NS separation parameters which avoid any collisions between NS and OS with probability 0.997 and 0.99.

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