

RADIONAVIGATION OF LOW ORBITAL CUBESAT AFTER SEPARATION FROM SOYUZ UPPER STAGE: ATTITUDE MOTION INFLUENCE

I.V. Belokonov,^{*} A.V. Kramlikh,[†] I.A. Timbai[‡]

The uncontrolled motion of CubeSat type nanosatellites (NS) around its mass center after separation from Soyuz upper stage is discussed. Two versions of motion are considered: the motion of spin-stabilized NS and the motion of aerodynamic stabilized NS. The stochastic model of initial conditions of attitude motion for NS is formulated. The possibilities of successful solutions of GPS/GLONASS navigation problem in the both modes are analyzed. The range of initial conditions of angle velocity for successful NS radionavigation is determined. The results can be used for formulation the performances of separation systems for CubeSat.

INTRODUCTION

The nanosatellite (NS) uncontrolled motion about the center of mass is considered. It is supposed that NS corresponds to the CubeSat2U standard (2 units of 10x10x20 cm in size, 2 kg in mass), it is dynamically symmetrical, the satellite density is permanent, and there is no orientation and stabilization system.

It is accepted that NS is launched from Soyuz upper stage booster making the uncontrolled motion after the separation of the main payload in a low roundabout orbit. This orbit is used for insertion of the "Progress" cargo spacecrafts with the maximum orbit height of 245 km and the minimum height 193 km. The upper stage after the payload separation gets some spinning about the longitudinal axis¹. The stochastic model of initial conditions of angular motion of upper stage is formulated¹.

Nanosatellite after separation from the stage will have some spinning about the longitudinal axis. It is considered two versions of motion of the NS around its mass center: the stabilization of the rotation when the statical stability factor of NS is zero, and aerodynamic stabilization, when the statical stability factor of NS is large enough.

In a stochastic setting of the possibility of solving navigation problems using signals GPS / GLONASS for the above modes of movement NS is analysed.

MOMENTS OF THE EXTERNAL FORCES ON THE NANOSATELLITE

Let's compare the value of the gravitational moment and restoring aerodynamic moment on

^{*} Professor, Dr. of Eng. Sci., Head of Space Research Department of the Samara State Aerospace University, Russia, E-mail: acad@ssau.ru

[†] Candidate of Eng. Sci., Samara State Aerospace University, Russia, E-mail: kramlikh@mail.ru.

[‡] Professor, Dr. of Eng. Sci., Samara State Aerospace University, Russia, E-mail: timbai@ssau.ru.

the nanosatellite. Magnetic moment will not be evaluated, assuming that it does not exceed the value of the gravitational moment. The influence of moment of dissipative aerodynamic forces is neglected. In the calculation of the restoring aerodynamic moment it was assumed that the free-flow NS and punch the gas molecules are inelastic. In this case, the aerodynamic force is the drag force that is determined by a cross-section NS to the flow direction.

Change of ratio of maximum value of the aerodynamic moment (for the statical stability factor $\Delta\bar{x} = \bar{x}_p - \bar{x}_g = 0.1$ and the drag force coefficient $C_{xa} = 2.7$) to maximum value of the gravitation moment from the altitude for two limiting values of atmospheric density² (night atmosphere at the minimal solar activity and day atmosphere at the maximum solar activity) are shown in Table 1.

Table 1. Change of ratio of maximum value of the aerodynamic moment to maximum value of the gravitation moment from the altitude.

Altitude, km	250	225	200	175	150	125
Night atmosphere at the minimal solar activity	60	150	380	1370	5990	43480
Day atmosphere at the maximum solar activity	340	530	900	1910	5640	43190

So the aerodynamic moment is the moment that defines the NS dynamics.

INITIAL CONDITIONS OF ANGULAR MOTION

Nanosatellite is launched from Soyuz upper stage, which performs uncontrolled motion after separating the primary payload. The rotary motion of the orbital stage is represented like regular precession at which the longitudinal axis passing through a mass center describes a circular cone about the kinetic moment vector (angular momentum) \vec{K} . This one saves the constant direction in the space (Figure 1), where α_k is the angle between the axis of symmetry and the vector \vec{K} (the half-angle cone of precession), $\dot{\psi}$ is the angular velocity of precession, $\dot{\phi}$ is the angular velocity of proper rotation¹.

Delay of separation of the NS from stage after payload separation t_d is 10-20 s. Assuming that the longitudinal axis of stage at the moment of separation of the payload coincides with the direction of its mass center velocity vector, then angle of attack α_0 (it is the angle between the longitudinal axis of stage and the mass center velocity vector) at the time of nanosatellite separation (Figure 1) can be defined by the formula

$$\alpha_0 = \arccos(\cos^2 \alpha_k + \sin^2 \alpha_k \cos \psi_0), \quad (1)$$

where $\cos \alpha_k = K_x / K$; $\dot{\psi} = K / J_n^{os}$; $\psi_0 = \dot{\psi} t_d$ is the angle precession at the time of NS separation; $K = \sqrt{K_x^2 + K_n^2}$ is the module of the kinetic moment of the stage; $K_x = J_x^{os} \omega_x^{os}$, $K_n = J_n^{os} \omega_n^{os}$ are longitudinal and transversal components of kinetic moment; J_x^{os} is the longitudinal moment of inertia of the stage; $J_y^{os} = J_z^{os} = J_n^{os}$ is the transversal moment of inertia of

the stage; $\omega_x^{os}, \omega_n^{os} = \sqrt{(\omega_y^{os})^2 + (\omega_z^{os})^2}$ are longitudinal and transversal components of angular velocity of the stage.

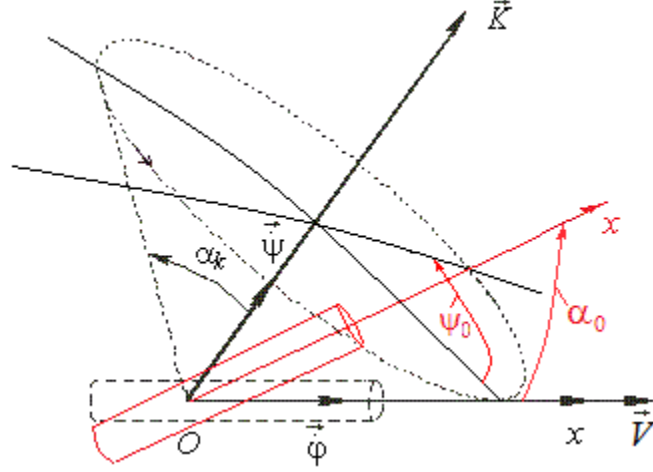


Figure 1. Initial conditions of angular motion.

For small values of the angle ψ_0 , assuming that $\cos\psi_0 \approx 1 - \psi_0^2/2$, on the basis of Equation (1) it can be obtained

$$\alpha_0 = \omega_n^{os} t_d. \quad (2)$$

Taking into account that the components of the transverse angular velocity are independent and normally distributed with variances $\sigma_{\omega_y^{os}}^2 = \sigma_{\omega_z^{os}}^2$ and zero mean, the value α_0 given by (2), has a Rayleigh distribution. The expressions for the probability density function and cumulative distribution function have the form:

$$f(\alpha_0) = \frac{\alpha_0}{\sigma_{\omega_y^{os}}^2 t_d^2} \exp\left(\frac{-\alpha_0^2}{2\sigma_{\omega_y^{os}}^2 t_d^2}\right), \quad (3)$$

$$F(\alpha_0) = 1 - \exp\left(\frac{-\alpha_0^2}{2\sigma_{\omega_y^{os}}^2 t_d^2}\right). \quad (4)$$

Figure 2 shows the empirical cumulative distribution function of the angle α_0 (step curve) and the analytical cumulative distribution function (4). Empirical cumulative distribution function obtained by statistical simulation (10000 numerical experiments) under ratio (1) with the distribution of the half-angle cone of precession α_k and the angular velocity of precession $\dot{\psi}^1$. The delay of separation of the nanosatellite from stage after payload separation was $t_d = 20$ s.

The mean and standard deviation of the angle α_0 obtained from the statistical simulation are

$\bar{\alpha}_0 = 20.9$ deg and $\sigma_{\alpha_0} = 10.8$ deg. In this case the mean and standard deviation of these values calculated using the formulas (3) and (4) are $\bar{\alpha}_0 = 20.9$ deg and $\sigma_{\alpha_0} = 10.9$ deg.

As seen from the above results, the values of statistical parameter derived from statistical simulation and calculated using (3) and (4) are virtually identical.

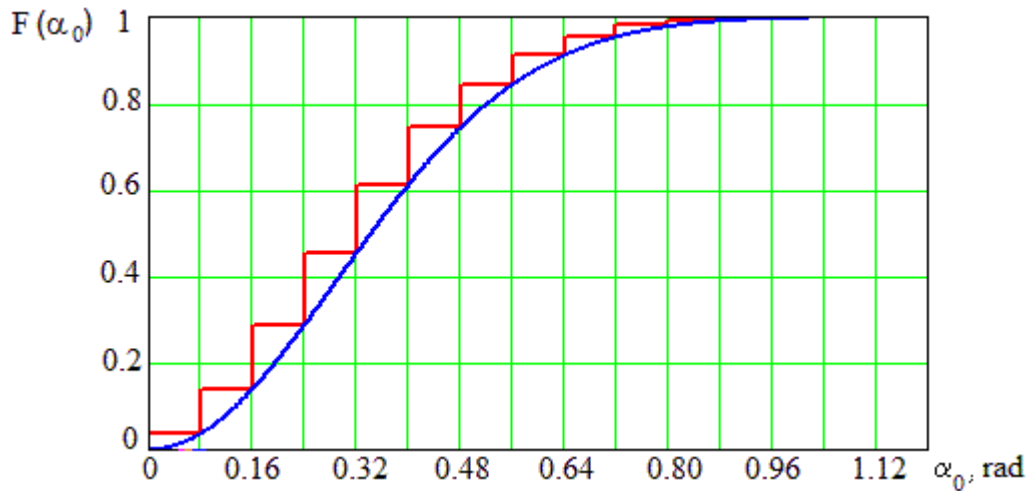
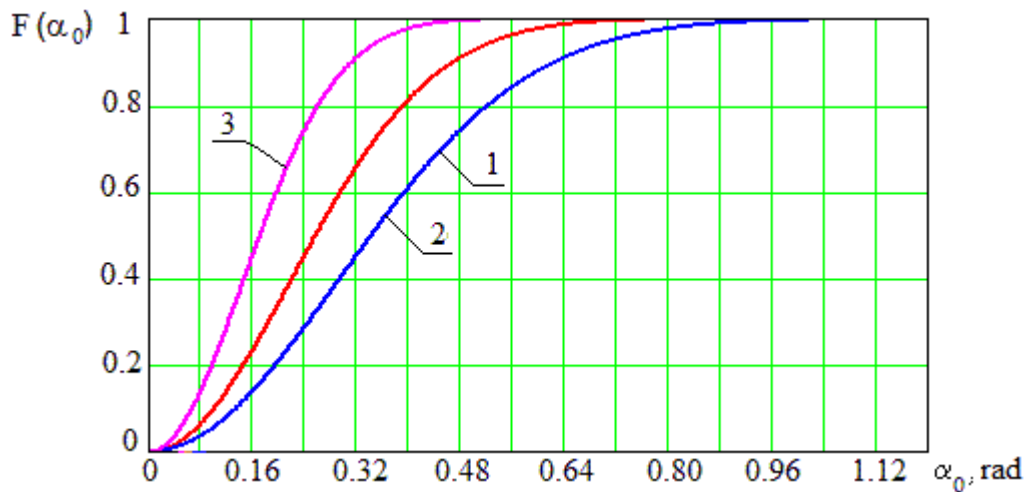


Figure 2. Cumulative distribution function of the angle of attack at the moment of nanosatellite separation.

Figure 3 shows the change of the cumulative distribution function of the angle of attack at the moment of NS separation α_0 from the time of delay NS separation from stage after payload separation.



**Figure 3. Change of the cumulative distribution function of the angle of attack at the moment of nanosatellite separation from the time of delay nanosatellite separation from stage:
1 — $t_d = 20$ s, 2 — $t_d = 15$ s, 3 — $t_d = 10$ s.**

SPIN-STABILIZED NANOSATELLITE

When the static stability factor of nanosatellite is small enough and can be neglected of the aerodynamic moment and other external moments, its rotational motion after separation from the stage is a regular precession. The values of the half-angle cone of precession α_k , the angular velocity of precession $\dot{\psi}$, the angular velocity of proper rotation $\dot{\phi}$ are calculated by formulas³

$$\alpha_k = \arctan(K_n / K_x),$$

$$\dot{\psi} = \frac{K}{J_n^{ns}}, \quad (5)$$

$$\dot{\phi} = \frac{J_n^{ns} - J_x^{ns}}{J_n^{ns}} |\omega_x^{ns}|,$$

where $K = \sqrt{K_x^2 + K_n^2}$ is the module of the kinetic moment of the nanosatellite; $K_x = J_x^{ns} \omega_x^{ns}$, $K_n = J_n \omega_n^{ns}$ are longitudinal and transversal components of kinetic moment; J_x^{ns} is the longitudinal moment of inertia of the NS, $J_y^{ns} = J_z^{ns} = J_n$ is the transversal moment of inertia of the NS; ω_x^{ns} , $\omega_n^{ns} = \sqrt{(\omega_y^{ns})^2 + (\omega_z^{ns})^2}$ are longitudinal and transversal components of angular velocity of the NS.

Suppose that the components of the transverse angular velocity are independent and normally distributed with zero mean and variance $\sigma_{\omega_y^{ns}}^2 = \sigma_{\omega_z^{ns}}^2 = \sigma^2$. Scatter of the longitudinal angular velocity ω_x^{ns} will be neglected. Then, calculating the distribution of function on distribution the argument⁴, we obtain analytical expressions for the probability density function and cumulative distribution function of the half-angle cone of precession $\alpha_k \in [0, \pi/2]$:

$$f(\alpha_k) = \frac{K_x^2 (\tan \alpha_k + \tan^3 \alpha_k)}{J_n^2 \sigma^2} \exp\left(\frac{-K_x^2 \tan^2 \alpha_k}{2J_n^2 \sigma^2}\right), \quad (6)$$

$$F(\alpha_k) = 1 - \exp\left(\frac{-K_x^2 \tan^2 \alpha_k}{2J_n^2 \sigma^2}\right)$$

and for the probability density function and cumulative distribution function of the angular velocity of precession $\dot{\psi} \in [K_x / J_n, \infty]$:

$$f(\dot{\psi}) = \frac{\dot{\psi}}{\sigma^2} \exp\left(\frac{K_x^2 - J_n^2 \dot{\psi}^2}{2J_n^2 \sigma^2}\right), \quad (7)$$

$$F(\dot{\psi}) = 1 - \exp\left(\frac{K_x^2 - J_n^2 \dot{\psi}^2}{2J_n^2 \sigma^2}\right).$$

Statistical simulation (10000 numerical experiments) has been carried out under formulas (5) for definition of statistical parameters of the distribution of the random variables of the half-angle cone of precession α_k , the angular velocity of precession $\dot{\psi}$ and the angular velocity of proper rotation $\dot{\phi}$. It was assumed that the angular velocity at the time of NS separation consists of angular velocity of stage and angular velocity caused an error of separation system of the NS. The components of the transverse angular velocity of the stage are independent and normally distributed with triple standard deviation $3\sigma_{\omega_y^{os}} = 3\sigma_{\omega_z^{os}} = 2.5$ deg/s and zero mean⁵. The longitudinal angular velocity of stage is normally distributed with triple standard deviation $3\sigma_{\omega_x^{os}} = 0.3$ deg/s, and the mean is 2.5 deg/s (Reference 5). It was assumed that the components of the angular velocity of the NS from separation system of the NS are independent and normally distributed with triple standard deviation $3\sigma_{\Delta\omega_y^{ns}} = 3\sigma_{\Delta\omega_z^{ns}} = 3$ deg/s, $3\sigma_{\Delta\omega_x^{ns}} = 0.6$ deg/s and zero mean. Longitudinal and transversal inertia moments of the NS were considered as random variables uniformly distributed within the ranges $(0.85J_x^{ns}; 1.15J_x^{ns})$ and $(0.85J_n; 1.15J_n)$.

Figures 4, 5 show the empirical cumulative distribution function of the half-angle cone of precession α_k , (step curve) and the angular velocity of precession $\dot{\psi}$ (step curve) respectively. The analytical cumulative distribution functions (6) and (7) are shown there ibid when the longitudinal and transverse moments of inertia are equal to their mean values.

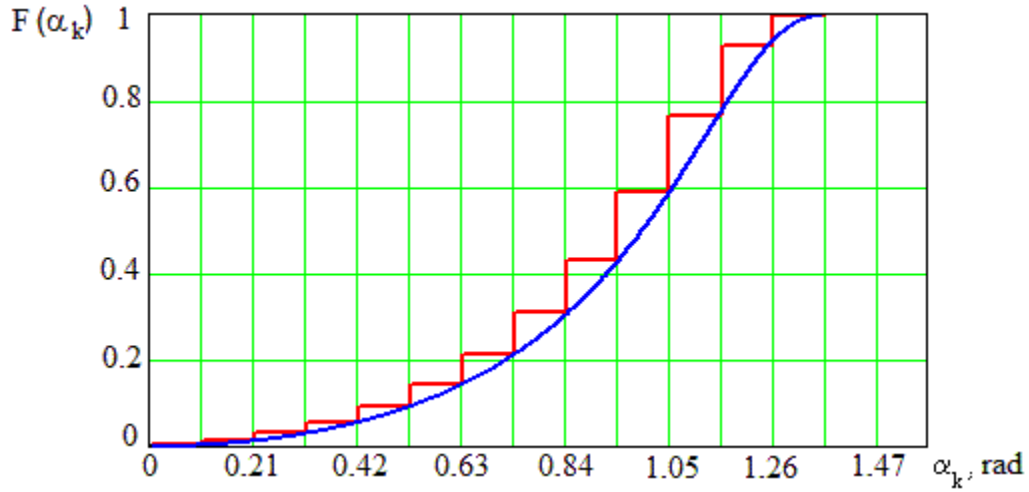


Figure 4. Cumulative distribution function of the half-angle cone of precession of the nanosatellite.

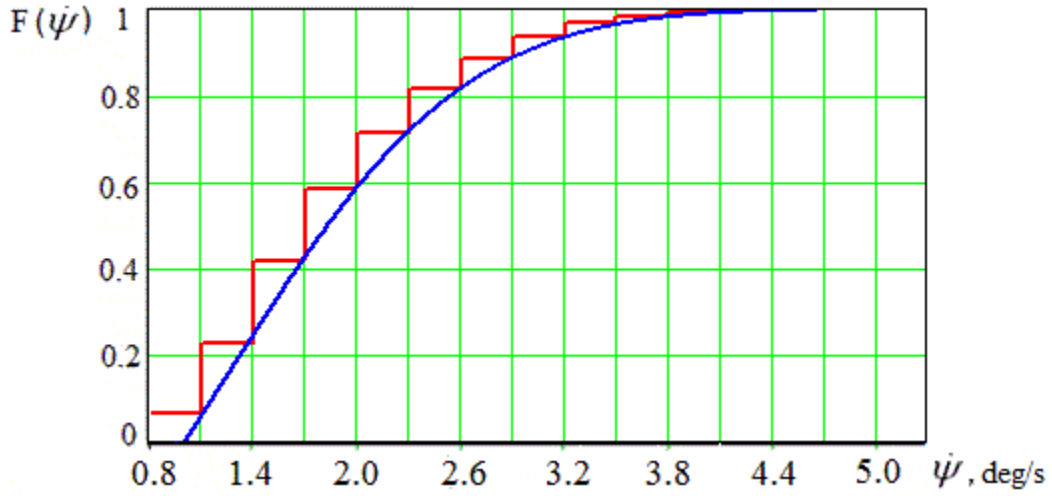


Figure 5. Cumulative distribution function of the angular velocity of precession of the nanosatellite.

Statistical parameters of the half-angle cone of precession α_k , the angular velocity of precession $\dot{\psi}$, the angular velocity of proper rotation $\dot{\phi}$ are obtained as a result of statistical simulation and calculated using expressions (6) and (7) are shown in Table 2 (\bar{x} is the mean and σ_x is standard deviation).

Table 2. Statistical parameters.

Value	Half-angle cone of precession		Angular velocity of precession		Angular velocity of proper rotation	
	$\bar{\alpha}_k$,	σ_{α_k} ,	$\bar{\dot{\psi}}$,	$\sigma_{\dot{\psi}}$,	$\bar{\dot{\phi}}$,	$\sigma_{\dot{\phi}}$,
Calculation	deg	deg	deg/s	deg/s	deg/s	deg/s
Statistical simulation	53.6	15.5	1.98	0.71	1.50	0.18
Formulas (6) and (7)	53.5	15.3	1.97	0.71	1.50	0

As seen from the above results, the values of statistical parameters derived from statistical simulation and calculated using (6) and (7) are virtually identical.

Figures 6 and 7 show the changes of the cumulative distribution functions of the half-angle cone of precession α_k and the angular velocity of precession $\dot{\psi}$ from value of the standard deviation of the components of the angular velocity of the NS from its separation system (curve 1 for $3\sigma_{\Delta\omega_y^{ns}} = 3\sigma_{\Delta\omega_z^{ns}} = 3$ deg/s, curve 2 for $3\sigma_{\Delta\omega_y^{ns}} = 3\sigma_{\Delta\omega_z^{ns}} = 2$ deg/s, curve 3 for $3\sigma_{\Delta\omega_y^{ns}} = 3\sigma_{\Delta\omega_z^{ns}} = 1$ deg/s, curve 4 characterizes the deviation of the cumulative distribution function for the case of ideal NS separation system from stage (absence the additional mistakes of initial angular velocity).

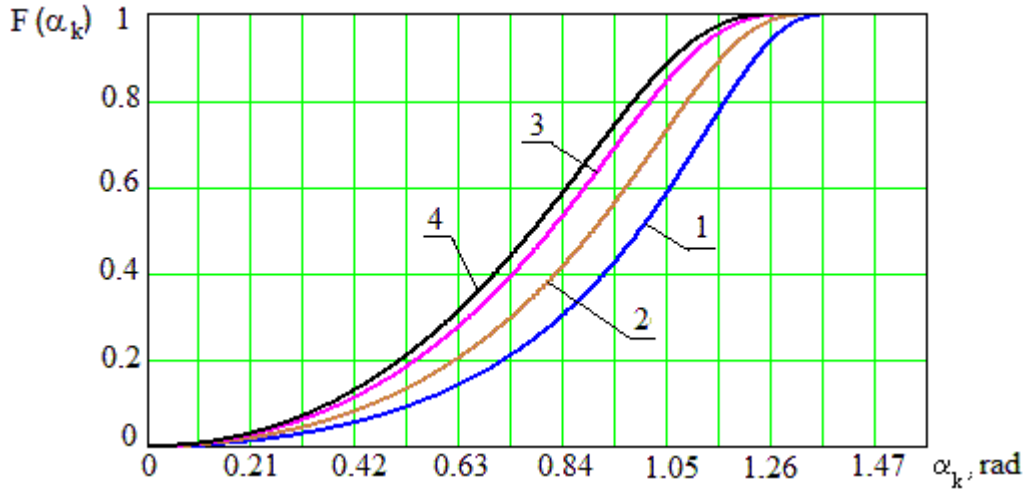


Figure 6. Cumulative distribution functions of the half-angle cone of precession of the nanosatellite.

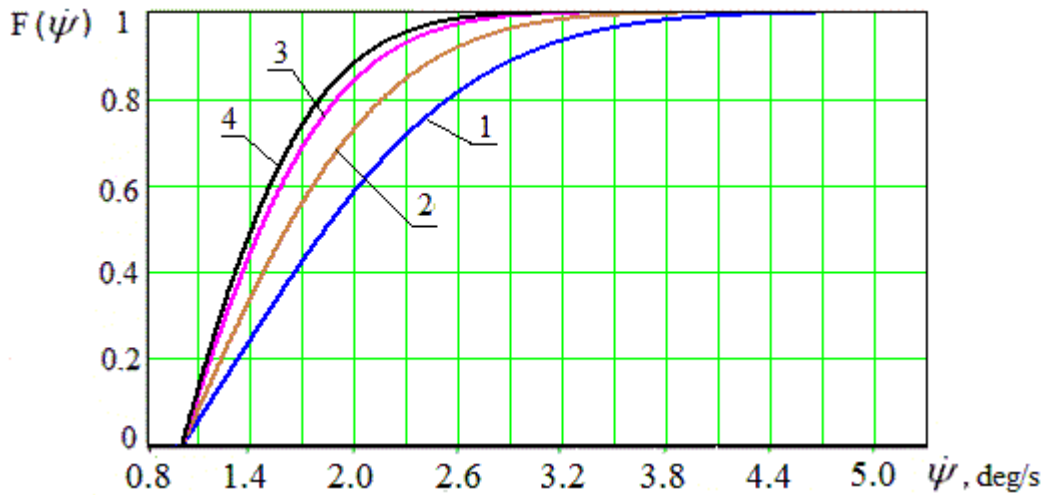


Figure 7. Cumulative distribution functions of the angular velocity of precession of the nanosatellite.

AERODYNAMIC STABILIZED NANOSATELLITE

If the static stability factor of the NS is large enough, the aerodynamic moment aspire to combine the longitudinal axis of the NS with the incoming airflow direction. However it's pitching motion is counteracted by gyroscopic forces leading to the forced precession of the kinetic moment vector about the mass center velocity vector. The kinetic moment vector deviates to the side of the vector of the aerodynamic moment. Neglecting the orbital angular velocity of the NS, averaging aerodynamic moment by the angle of attack and approximating sinusoidal angle of attack

dependence, change of the angle of attack NS can be described by the following equation⁶

$$\ddot{\alpha} + (G - R \cos \alpha)(R - G \cos \alpha) / \sin^3 \alpha + a \sin \alpha = 0, \quad (8)$$

where $R = J_x^{ns} \omega_x^{ns} / J_n = \text{const}$, $G = R \cos \alpha + (-\omega_y^{ns} \cos \varphi_n + \omega_z^{ns} \sin \varphi_n) \sin \alpha = \text{const}$ are projections of a kinetic moment vector on the longitudinal axis of the NS and on the mass center velocity vector, normalized with respect to the transversal moment of inertia, φ_n is the aerodynamic roll angle (the angular velocity of proper rotation); $a = -m_\alpha S l V^2 \rho(H) / (2J_n)$, m_α is the coefficient of restoring moment measured about the center of mass of the NS ($m_\alpha = -0.7$ for the statical stability factor $\Delta \bar{x} = 0.1$), $S = 0.01 \text{ m}^2$ is the characteristic area of the NS, $l = 0.2 \text{ m}$ is the characteristic dimension of the NS, V is the mass center velocity, $\rho(h)$ is the atmospheric density for the flight altitude H .

Precession of the longitudinal axis of the NS about the mass center velocity in the time interval equal to the period of a complete revolution and the opposite direction to a given vector is called "inverse" precession, and coinciding with the direction of the velocity vector of the center of mass is called a "direct" precession⁷. Under the condition $R > G$ is implemented the "inverse" precession, when the condition $G > R$ is implemented the "direct" precession (see Figure 8).

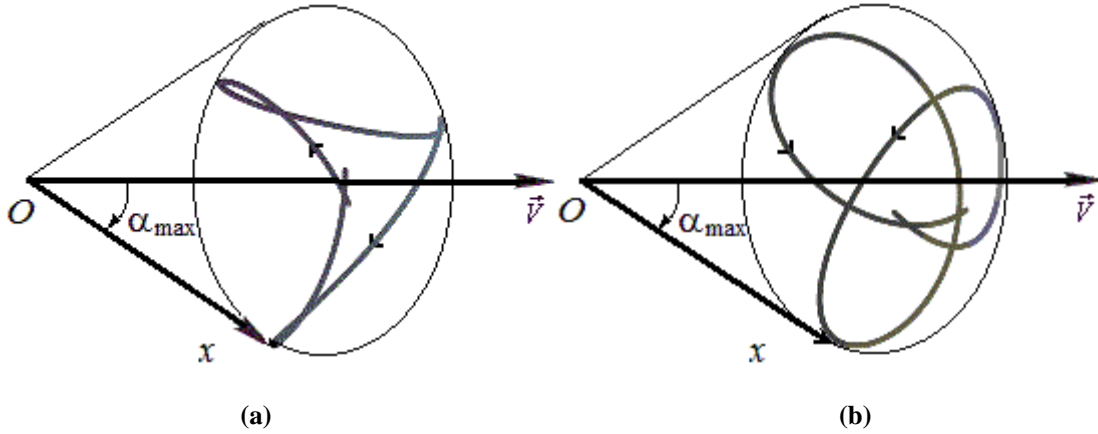


Figure 8. Trajectory of the end of the longitudinal axis of the orbital stage on the unit sphere concerning the trajectory reference frame: (a) - "inverse" precession, (b) - "direct" precession.

The integral of energy of system (8) for constant H has the following form

$$\dot{\alpha}^2 / 2 + (R^2 + G^2 - 2RG \cos \alpha) / (2 \sin^2 \alpha) - a \cos \alpha = E. \quad (9)$$

The value of E is determined by the initial conditions, with $\dot{\alpha}_0 = \omega_y^{ns} \cos \varphi_n + \omega_z^{ns} \sin \varphi_n$. The maximum value of the angle of attack is determined by the equation

$$(R^2 + G^2 - 2RG \cos \alpha_{\max}) / (2 \sin^2 \alpha_{\max}) - a \cos \alpha_{\max} - E = 0 \quad (10)$$

For determination the statistical parameters of the distribution of the maximum angle of attack

of the NS out a statistical simulation has been carried under ratio (9), (10), with (1). The delay of separation of the NS from stage after payload separation was $t_d = 15$ s. The altitude of separation of the NS from stage is $H = 200$ km. The values of atmospheric density were taken appropriate night atmosphere at the minimal solar activity².

Figures 9 and 10 show the changes of the cumulative distribution functions of the maximum angle of attack of the NS (for the static stability factors $\Delta\bar{x} = 0.1$ and $\Delta\bar{x} = 0.15$) from value of the standard deviation of the components of the angular velocity of the NS from its separation system (curve 1 for $3\sigma_{\Delta\omega_y^{ns}} = 3\sigma_{\Delta\omega_z^{ns}} = 3$ deg/s, $3\sigma_{\Delta\omega_x^{ns}} = 0.6$ deg/s; curve 2 for $3\sigma_{\Delta\omega_y^{ns}} = 3\sigma_{\Delta\omega_z^{ns}} = 2$ deg/s, $3\sigma_{\Delta\omega_x^{ns}} = 0.4$ deg/s; curve 3 for $3\sigma_{\Delta\omega_y^{ns}} = 3\sigma_{\Delta\omega_z^{ns}} = 1$ deg/s, $3\sigma_{\Delta\omega_x^{ns}} = 0.2$ deg/s; curve 4 characterizes the change of the cumulative distribution function for the case of ideal NS separation system from stage (absence the additional mistakes of initial angular velocity).

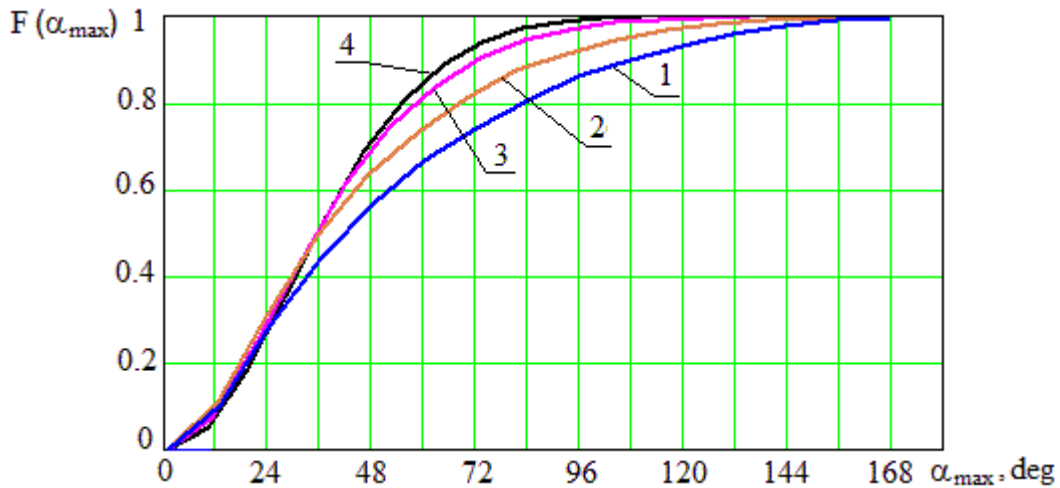


Figure 9. Cumulative distribution function of the maximum angle of attack of the nanosatellite (for the static stability factor is $\Delta\bar{x} = 0.1$).

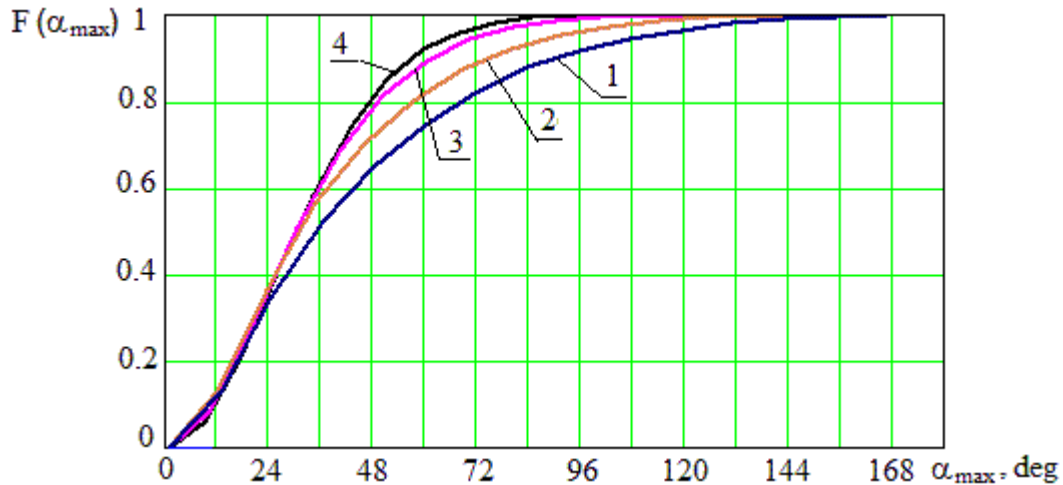


Figure 10. Cumulative distribution function of the maximum angle of attack of the nanosatellite (for the static stability factor is $\Delta\bar{x} = 0.15$).

RADIONAVIGATION OF NANOSATELLITE

The analysis of possibility of navigation task solution for the NS uncontrolled motion was made in accordance with mentioned above probabilistic models.

The following definitions were used:

- the "full" navigational problem is named the determination of three coordinates (x, y, z) and time reference (t). The solution of this task is possible under condition of continuous visibility of the same group consisting of not less, than four navigation satellites;

- the "limited" problem of navigation is named the determination of three coordinates (x, y, z) without time reference. The solution of this task is possible under condition of continuous visibility of the same group of three navigation satellites. At smaller number of continuous visible satellites the solution of a navigation task is not possible;

- "hot", "warm" and "cold" modes are understood as the solutions of a navigation problem at duration of an interval of visibility of the same group of navigational satellites not less than 30, 90 and 180 seconds, respectively.

Analysis of possibility of navigation task resolution was made on a model using following initial conditions:

- errors of angular velocity (3σ) of the NS separation from upper stage are assumed equal 0, 1, 2, 3 deg/s;

- the NS orbit is near-circular orbit with 200 km average altitude and $i = 51$ deg orbit inclination;

- in the case of spin-stabilized NS the angle of the NS visibility cone from the phase centre of the antenna δ_c is determined over the angle α_k and equal to $\delta_c = 180^\circ - 2\alpha_k$. Values of the angle α_k are taken with accordance to the theoretical law of distribution function as described by (6);

– in the case of aerodynamic stabilized NS the angle of the NS visibility cone from the phase centre of the antenna δ_c is determined over the angle α_{\max} and equal to $\delta_c = 180^\circ - 2\alpha_{\max}$. Values of the angle α_{\max} are taken in accordance with the cumulative distribution function is shown in Figures 9 and 10;

- the vector of the antenna phase centre is oriented along the longitudinal axis of the NS;
- the time interval is equal to the NS orbital motion period.

For calculation of the probability of solution of the navigation problem was used a formula of total probability was used⁴:

$$P(A) = \sum_{i=1}^n P(A | \delta_c^i) P(\delta_c^i),$$

where A is the event of probability of navigation problem solution, $P(A | \delta_c^i)$ is the conditional probability of navigation problem solution for fixed angle δ_c , $P(\delta_c^i)$ is the probability of realization of the angle δ_c , n is the quantity of angle values δ_c .

The probability of possible solutions for "limited" and "full" navigation problems in the "hot", "warm" and "cold" modes for the case of spin-stabilized NS is shown at Figures 11 and 12. As seen, the probability $P(A)$ for solution of the "limited" problem of navigation at the maximum deviations of the separation initial conditions is 0.91, and the probability for solution of the "full" navigation problem is 0.77.

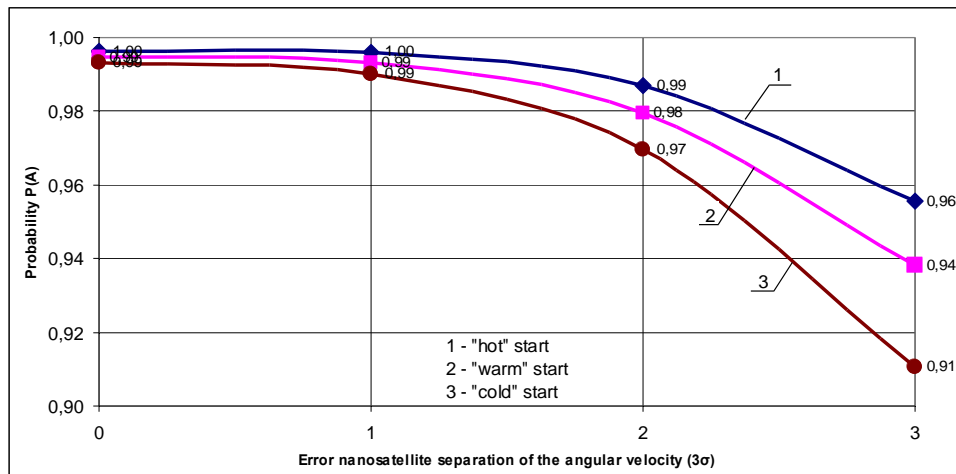


Figure 11. Probability of the "limited" navigation problem solving (spin-stabilized NS).

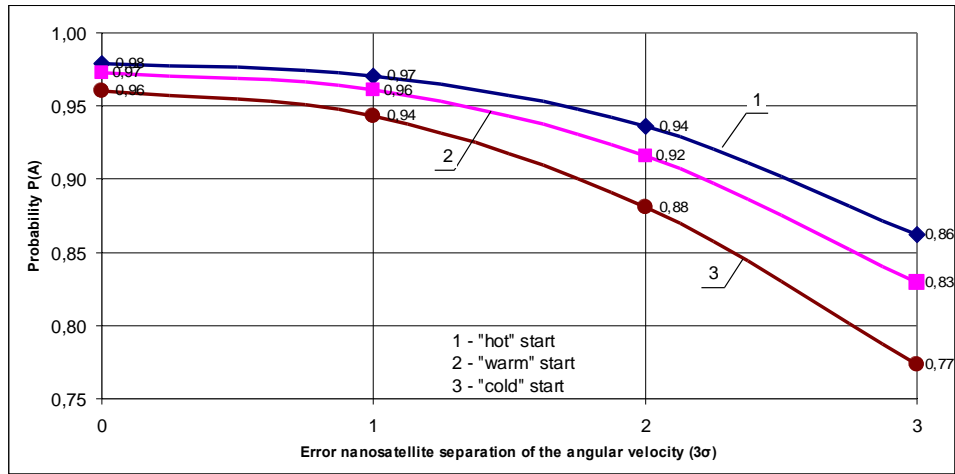


Figure 12. Probability of the "full" navigation problem solving (spin-stabilized NS).

The probability of possible solutions for "limited" and "full" problems of navigation in the "hot", "warm" and "cold" modes for the case of aerodynamic stabilisation for various values of static stability factor is shown at Figures 13-16. As seen, the probability $P(A)$ for solution both of the "limited" problem of navigation and of the "full" problem of navigation at the maximum deviations of the initial separation conditions is 0.74 in case when the static stability factor is $\Delta\bar{x} = 0.1$ and not less than 0.82 in case is $\Delta\bar{x} = 0.15$.

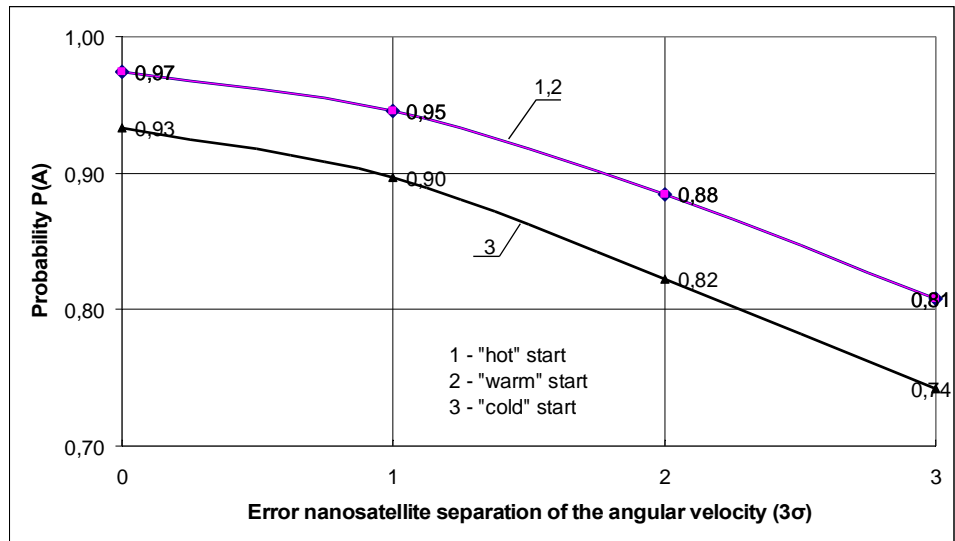


Figure 13. Probability of the "limited" navigation problem solving (aerodynamic stabilized NS, static stability factor is $\Delta\bar{x} = 0.1$).

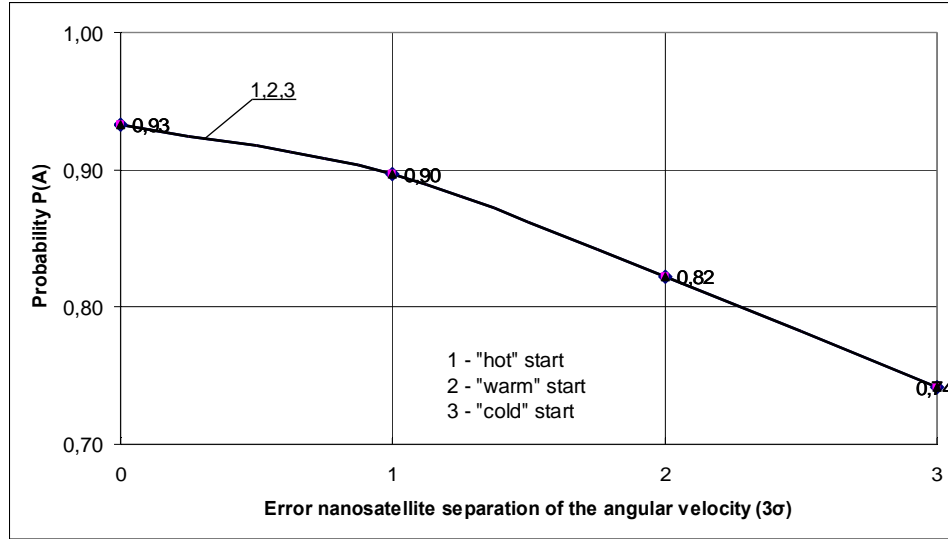


Figure 14. Probability of solving the "full" navigation problem for different values of δ_c angle (aerodynamic stabilized NS, statical stability factor is $\Delta\bar{x} = 0.1$).

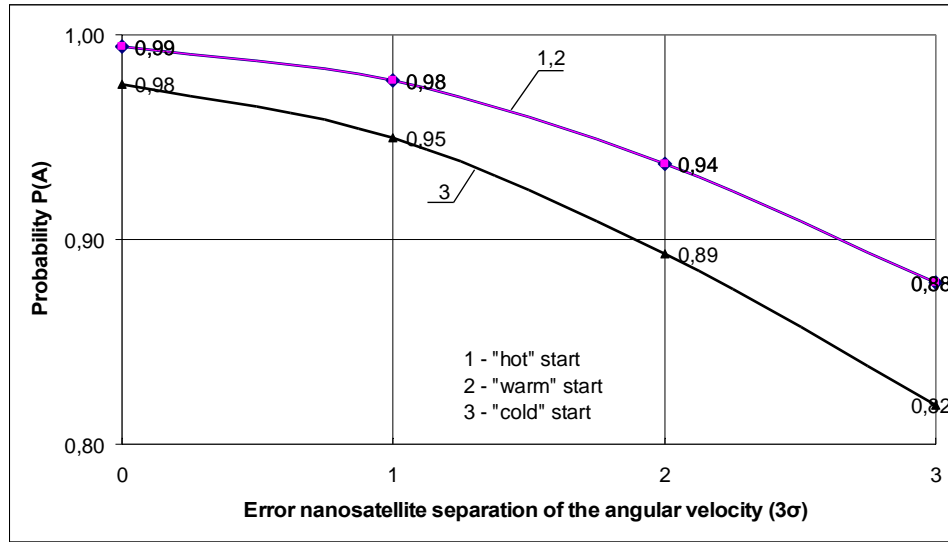


Figure 15. Probability of the "limited" navigation problem solving (aerodynamic stabilized NS, statical stability factor is $\Delta\bar{x} = 0.15$).

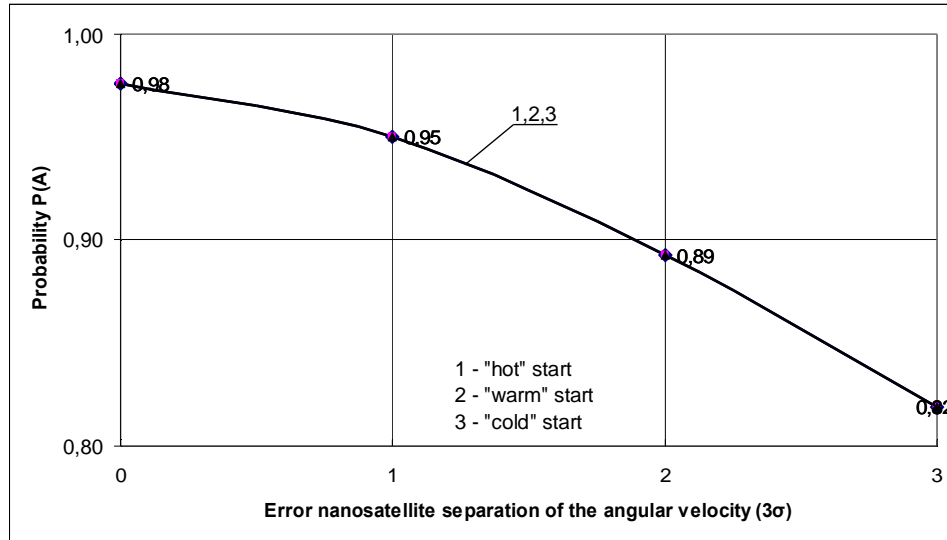


Figure 16. Probability of solving the "full" navigation problem for different values of δ_c angle (aerodynamic stabilized NS, static stability factor is $\Delta\bar{x} = 0.15$).

That's why the NS separation error in the angular velocity has not exceed 1 deg/s for aerodynamically stabilized NS with the value of the static stability factor $\Delta\bar{x} = 0.1$ if navigational problem have to be solved with a probability of at least 0.9 during one orbital period. If the static stability factor is increased up to $\Delta\bar{x} = 0.15$ the probability of solving of the navigation problem is also increased up to 0.95.

CONCLUSIONS

The stochastic models of the initial conditions of attitude motion for nanosatellite are formulated for two cases of motion: the motion of spin-stabilized nanosatellite and the motion of aerodynamic stabilized nanosatellite. The possibility of the successful solution of the navigation problem by GPS/GLONASS signals for these modes of movement and different values of the static stability factors of nanosatellite for various separation system errors is proved. The obtained results allow to generate the angular velocity errors limitations for separation systems of nanosatellite. It is shown that the increasing of the static stability factor reduces the requirements for the level of permissible disturbance from separation systems of nanosatellite, and tightening of the allowable level of disturbances from the separation system to increase the probability of solution of navigation problem.

ACKNOWLEDGEMENT

The reported study was partially supported by RFBR, research project No. 13-08-97015-r_Volga_region_a.

REFERENCES

- ¹ I. Belokonov, A. Storozh, I. Timbay Modes of motion of Soyuz orbital stage after payload separation at carrying out of the short-term research experiments, *Advances in the Astronautical Sciences*, 2012, Vol. 145, pp. 99-107.
- ² G.S. Narimanov, M.K. Tikhonravov, *Fundamentals of the theory of flight spacecraft* [in Russian], Mashinostroenie, Moscow, 1972.
- ³ A.P. Markeev, *Theoretical Mechanics* [in Russian], Nauka, Moscow, 1990.
- ⁴ E.C. Wentzel, *Probability theory* [in Russian], Fizmatgiz, Moscow, 1958.
- ⁵ I.V. Belokonov, G.E. Kruglov, V.I. Trushlyakov, V.V. Yudintsev V.V. “Analysis of Possibility of Residual Propellant Utilization in Soyuz Orbit Stage for Controllable Descent” [in Russian], *Vestnik SSAU*, 2010, 2 (22), pp. 105 – 111.
- ⁶ V.A. Yaroshevsky, *Motion of an uncontrolled body in the atmosphere* [in Russian], Mashinostroenie, Moscow, 1978.
- ⁷ I. D.H. Platus, Angle of attack convergence windward meridian rotation rate of rolling re-entry vehicles, *AIAA Journal*, 1969, Vol.7, pp. 2324-2330.