Approach for estimation of nanosatellite's motion concerning of mass centre by trajectory measurements

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Abstract: The restore of the SamSat-218D nanosatellite movement by trajectory measurements is analyzed. Experimentally were confirmed features of the behavior of nanosatellites in low orbits, due to both the influence of the atmosphere and their inherent mass-inertial characteristics: the lifetime of nanosatellites is less, and the angular acceleration generated by the aerodynamic moment is much higher than that of satellites with large sizes and masses. The change in the ballistic coefficient in time is estimated from known trajectory measurements and information on the average density of the atmosphere at the points of trajectory measurements. The ballistic coefficient of the SamSat-218D nanosatellite, having the shape of a rectangular parallelepiped, depends on the spatial angle of attack and the angle of proper rotation. The ratio of the maximum value of the ballistic coefficient to the minimum value is 4.75. This made it possible to evaluate the nature of the possible motion relative to the center of mass of the nanosatellite by the nature of the change in the ballistic coefficient. The most probable motion relative to the center of mass of the SamSat-218D nanosatellite is the transitional mode of motion between different equilibrium positions, due to commensurate aerodynamic and gravitational moments and insignificant angular velocities.

1. INTRODUCTION

Since 2014, nanosatellites have been developed at the Samara University. The SamSat-218D nanosatellite [1] of the CubeSat 3U format is the first nanosatellite developed by students and scientists of Samara University. On April 28, 2016, it became a participant of the first launch campaign from the Vostochny cosmodrome, and simultaneously with two other satellites (MVL-300 and Aist-2D) was launched into orbit with an inclination of 97.3 ° and an average height of 486 km using the Soyuz 2.1a launch vehicle. SamSat-218D was designed to improve number of technological and educational tasks. First of all, it was intended for testing the orientation control algorithms of nanosatellites. However, since launching into orbit, it was not possible to establish communication with nanosatellite.

Some features of the motion of nanosatellites in low orbits are described in [2].

1) The value of the ballistic coefficient of a nanosatellite is more than that of a satellite with large dimensions and mass (with the same bulk density), which leads to a decrease in its lifetime in orbit. It makes possible, given the small planned duration of the active work of the nanosatellite (usually from six months to a year), to use low orbits effectively and to avoid clogging of near-Earth space.

2) The angular acceleration of a nanosatellite due to the aerodynamic moment is much more than that of a satellite with large dimensions and mass (for the same values of the relative static stability margin and bulk density). This expands the range of heights at which the aerodynamic moment acting on the nanosatellite is significant and can be used to passively stabilize along the velocity vector of the center of mass.

3) The value of the ballistic coefficient of a nanosatellite substantially depends on its orientation. The following formula expresses the relationship between the ballistic coefficient of a CubeSat nanosatellite and its orientation [3]

$$\sigma(\alpha,\varphi)=c_0\tilde{S}(\alpha,\varphi)S/m,$$

where α – angle of attack; φ – angle of proper rotation; c_0 – the drag coefficient, which can take values from 2 to 3 depending on the physical properties of the gas and the surface of the nanosatellite (for design studies is assumed to be 2.2); S – characteristic area; m – nanosatellite mass; $\tilde{S}(\alpha,\varphi)$ – the projection area of a CubeSat nanosatellite on a plane perpendicular to the velocity vector, divided to the characteristic area:

$$\tilde{S}(\alpha,\varphi)=(|\cos\alpha|+k_s\sin\alpha(|\sin\varphi|+\cos|\varphi|))/S,$$

where k_s – the ratio of the area of one of the side surfaces to the characteristic area.

To analyze the angular motion of a nanosatellite, for the case when the angular velocity of its own rotation is close to uniform, the expression for the ballistic coefficient can be averaged by the angle of proper rotation

$$\sigma(\alpha)=c_0(|\cos\alpha|+\frac{4k_s}{\pi}\sin\alpha)S/m.$$

Figure 1 shows a graph of the dependence of the SamSat-218D nanosatellite ballistic coefficient on the angle of attack for different values of the anglφe of proper rotation. The ratio of the maximum ballistic coefficient to the minimum is 4.75. This fact makes it possible to extract information on the orientation and dynamics of the movement of a nanosatellite from information on the value of a ballistic coefficient.

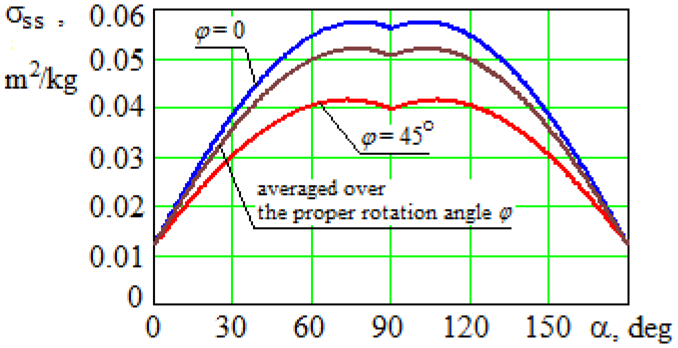


Fig.1. Dependence of the SamSat-218D nanosatellite ballistic coefficient on the angle of attack α and proper rotation angle φ

2. FORMULATION OF THE PROBLEM

Let's look at the trajectory measurements of the SamSat-218D nanosatellite. Figure 2 shows the changes in the height of its orbit. The information is based on the processing of data from the NORAD TLE files [4].

It should be noted that the considered time interval of the satellite (from April 28, 2016 till November 24, 2018) corresponds to a decrease of solar activity, which leads to a decrease in the density of the upper atmosphere and the height reduction rate.

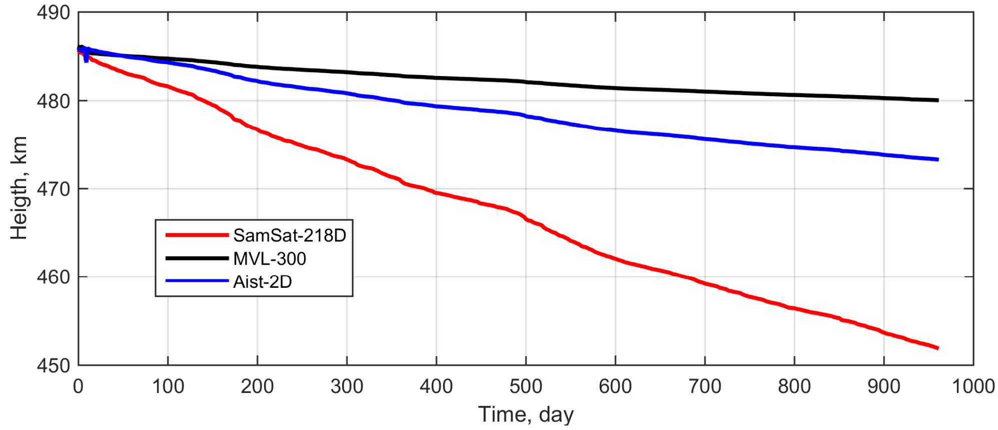


Fig.2. Changing the height of the orbits of the SamSat-218D, small satellites MVL-300 and Aist-2D

Using the information on the SamSat-218D nanosatellite height changing (trajectory measurements), as well as data on its design parameters (mass, inertia moments, aerodynamic characteristics) and the results of theoretical studies of the CubeSat3U nanosatellite movement dynamics [3] possible to solve two important tasks:

1) to study the changes in the ballistic coefficient of the SamSat-218D nanosatellite;

2) to identify the most probable mode of movement of the SamSat-218D nanosatellite relative to the center of mass, which was implemented after separation from the booster "Volga".

3. THE METHOD OF ESTIMATING THE BALLISTIC COEFFICIENT

Satellite orbit perturbations caused by the action of aerodynamic acceleration Φ, for a circular orbit, are described by the well-known formula [5]:

$$\dot{r} = 2r\sqrt{\frac{r}{\mu}}\Phi \ , \qquad (1)$$

where r – current radius vector; μ – Earth gravitational parameter; $\Phi = -\sigma q$ – disturbing aerodynamic acceleration; $q = \rho V^2 / 2$ – velocity head; ρ – atmosphere density; $V = \sqrt{\mu / r}$ – flight speed.

Using this formula, as well as the TLE files for SamSat-218D and Aist-2D satellites, the following method is proposed for calculating the average daily ballistic coefficient for the SamSat-218D nanosatellite:

1) calculation of the radius-vectors from the data of TLE files for SamSat-218D and Aist-2D satellites

$$r = \sqrt[3]{\mu / n^2},$$

where n – mean motion;

2) smoothing and re-discretization of the radius-vector data table of both satellites, by a cubic smoothing spline with a one-day sampling step and a smoothing parameter $p = 0.95$ (the selected value of the smoothing parameter provides an acceptable interpolation with the removal of high-frequency noise);

3) calculation of the derivative of the radius vector for SamSat-218D and Aist-2D by the method of numerical differentiation;

4) determination of the average daily density of the atmosphere (Figure 3a) via known deceleration of the Aist-2D satellite, for which the magnitude of the ballistic coefficient is known and equals $\bar{\sigma}_{const} = 0.0227 \ m^2 / kg$ (Aist-2D maintains its orientation in the orbital coordinate system)

$$\bar{\rho}_a = -\dot{r}_a / \bar{\sigma}_{const} \sqrt{\mu r_a},$$

where r_a – radius vector of Aist-2D satellite; $\dot{r}_a$ – derivative of the radius vector of Aist-2D satellite;

5) calculation of the average daily atmospheric density for the SamSat-218D satellite using the correlation formula for the upper atmosphere layers from GOST 25645.101-83 [6] (Figure 3a)

$$\bar{\rho}_{ss} = \bar{\rho}_a \cdot \exp\left(a_2 \left[\sqrt{r_a - a_3} - \sqrt{r_{ss} - a_3}\right]\right),$$

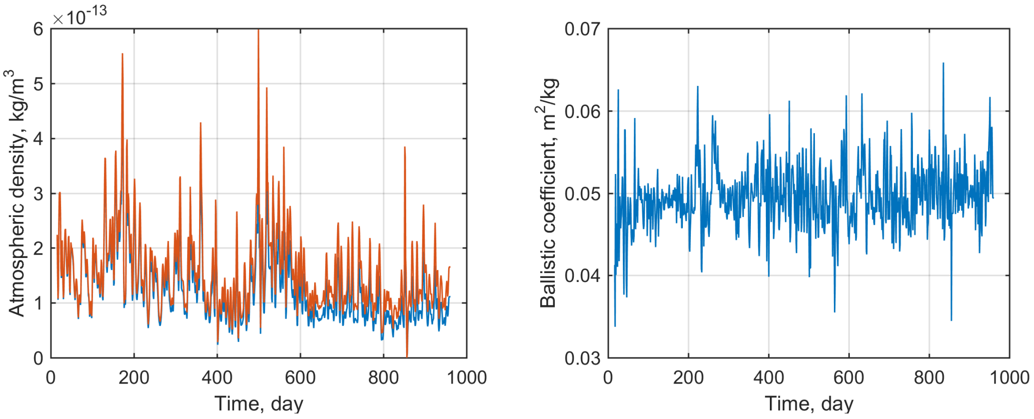
where coefficients $a_2 = 0.71604 \ km^{-1/2}$ and $a_3 = 6461.34 \ km$ are taken for altitude range $180 \ km \le h < 600 \ km$ and solar activity index $F_{10.7} \approx 75 \cdot 10^{-22} \ W / \left(m^2 \cdot Hz\right)$;

6) calculation of the daily averaged ballistic coefficient $\bar{\sigma}_{ss}$ for SamSat-218D:

$$\bar{\sigma}_{ss} = -\dot{r}_{ss} / \bar{\rho}_{ss} \sqrt{\mu r_{ss}}.$$

r_{ss} –radius vector of SamSat-218D satellite; $\dot{r}_{ss}$ –radius vector derivative for SamSat-218D satellite.

Figure 3b presents the results of estimating the averaged ballistic coefficient for the SamSat-218D nanosatellite.



(a) (b)

Fig.3. (a) Changes in atmospheric density over time for the Aist-2D satellites (blue), SamSat-218D (red); (b) changes in averaged per day ballistic coefficient of SamSat-218D nanosatellite

Figure 4 shows a fragment of the graph of the change of the ballistic coefficient averaged per day for the time period from 16 to 100 days. It should be taken into account that variations in the averaged ballistic coefficient are caused by both errors of trajectory measurements and the nature of the angular motion of the nanosatellite.

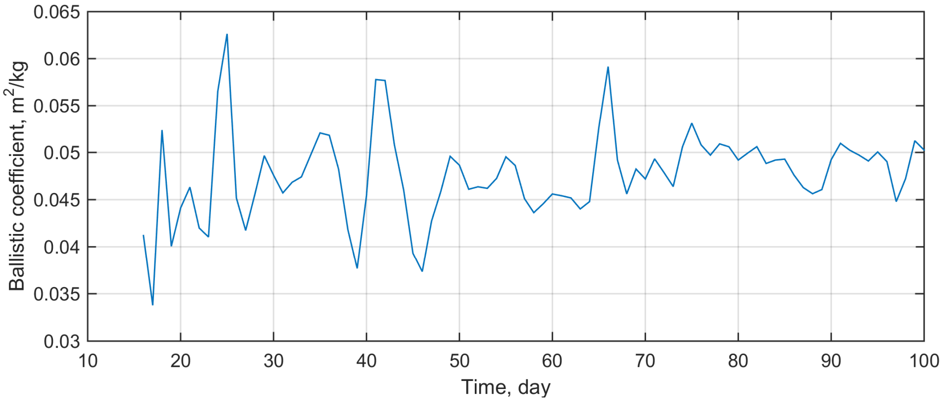


Fig.4. The change of averaged per day ballistic coefficient (from 16 to 100 days)

4. MOTION ANALYSIS

For qualitative analysis of the motion relative to the center of mass of the SamSat-218D nanosatellite, an approximate model of angular motion in the plane of a circular orbit with respect to the trajectory coordinate system is used. The model describes the change in the angle of attack under the action of the gravitational moment and the aero-dynamic recovery moment [3]:

$$\ddot{\alpha} - a(H)\sin\alpha - c(H)\sin 2\alpha = 0\,, \qquad (2)$$

where $a(H) = a_0 Slq(H)/J_n$ – coefficient characterized the aerodynamic recovery moment; a_0 – the coefficient for approximation by sinusoidal dependence of the coefficient of the aerodynamic recovery moment; l – the characteristic length of the nanosatellite; H – flight altitude; $c(H) = 3(J_n - J_x)(\omega(H))^2/(2J_n)$ – the coefficient characterized the action of the gravitational moment; $\omega(H) = \sqrt{\mu/(R_3 + H)^3}$ – the angular velocity concerning the nanosatellite mass center; R_3 – Earth radius.

The change in the height of a circular orbit due to atmospheric drag occurs very slowly, and when considering the angular motion of a nanosatellite on one or several turns, it can be accepted $H = const$. In this case, for system (2) the energy integral is

$$\dot{\alpha}^2/2 + a\cos\alpha + c\cos^2\alpha = E_0 \tag{3}$$

where $E_0 = a\cos\alpha_0 + c\cos^2\alpha_0 + \dot{\alpha}_0^2/2$ – determined by initial conditions.

The nature of the movement of the nanosatellite is determined by the ratio of a, c and E_0. In the case of $a < 0, c > 0$, there are two kinds of phase portraits.

1) $|a| \geq 2c$ (gravitational moment less than aerodynamic one). The phase portrait is similar to the oscillatory system of the pendulum type. In this case, the nanosatellite has two equilibrium positions for the angle of attack – stable with $\alpha = 0 + 2n\pi$ $(n = 0 \pm 1, \pm 2, ...)$ and unstable with $\alpha = \pi + 2n\pi$ $(n = 0 \pm 1, \pm 2, ...)$. The rotational motion mode of the nanosatellite corresponds to the condition: $E_0 > -a + c$, oscillatory motion mode relative to stable equilibrium $\alpha = 0 + 2n\pi$ $(n = 0 \pm 1, \pm 2, ...)$ corresponds to the condition: $E_0 < -a + c$. The areas of possible motions are separated by a separatrix.

2) $c > 0.5|a|$ (gravitational moment greater then aerodynamic one). With this ratio, there are four areas of motion of the nanosatellite: a rotational region and three oscillatory regions (a schematic view of the phase portrait is shown in Figure 5). The nanosatellite has four equilibrium positions for the angle of attack: $\alpha_* = \pm\arccos(-0.5a/c) + 2n\pi$ $(n = 0 \pm 1, \pm 2, ...)$, $\alpha = 0 + 2n\pi$ $(n = 0 \pm 1, \pm 2, ...)$, $\alpha = \pi + 2n\pi$ $(n = 0 \pm 1, \pm 2, ...)$. The rotational motion mode of the nanosatellite corresponds to the condition: $E_0 > -a + c$ – phase trajectory 1, oscillatory motion mode relative to the equilibrium position $\alpha = 0 + 2n\pi$ $(n = 0 \pm 1, \pm 2, ...)$ corresponds to the condition: $-a + c < E_0 > a + c$ – phase trajectory 2, oscillatory motion mode relative to the equilibrium position α_* corresponds to the condition: $E_0 < a + c$ – phase trajectory 3. The areas of possible motions modes are separated by separatrixes (phase trajectories 4 and 5).

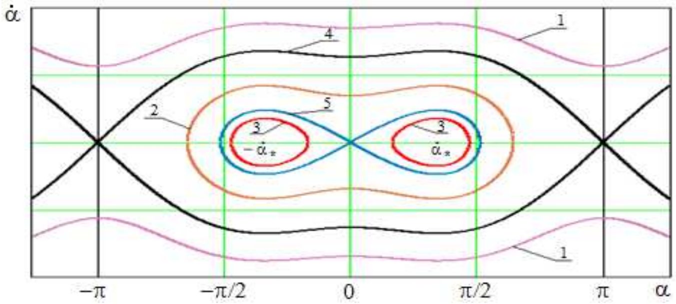


Fig.5. Possible phase portraits

Due to the uncertainty of the initial angular velocities obtained by the SamSat-218D nanosatellite after separation, multiple motion simulations were carried out in a wide range of initial angular velocities, using data on changes in the atmospheric density (Figure 3a), for ensuring compliance with the found trajectory measurements of variations in the average ballistic coefficient (Figure 3b).

Figure 6 shows the change in the ratio of the maximum values of the aerodynamic and gravitational moments for the time period from days 16 to 100, which caused by the cyclical nature of changes in the density of the atmosphere by solar activity.

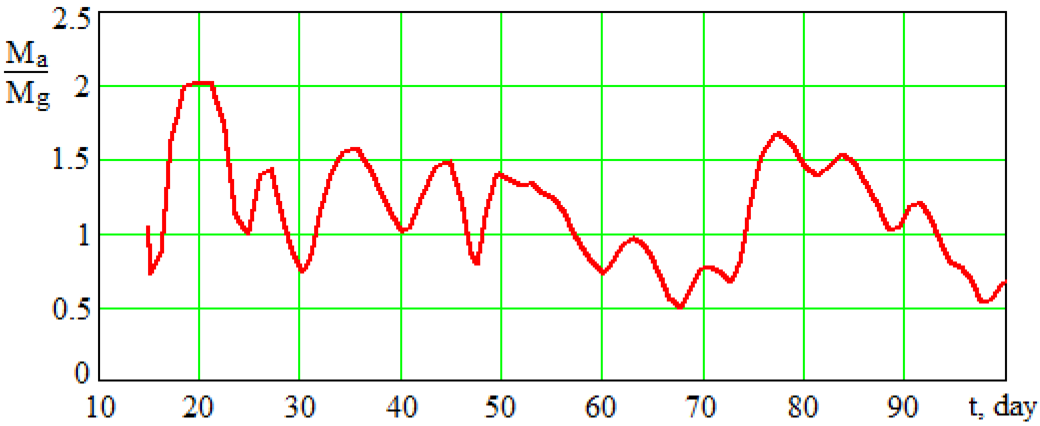
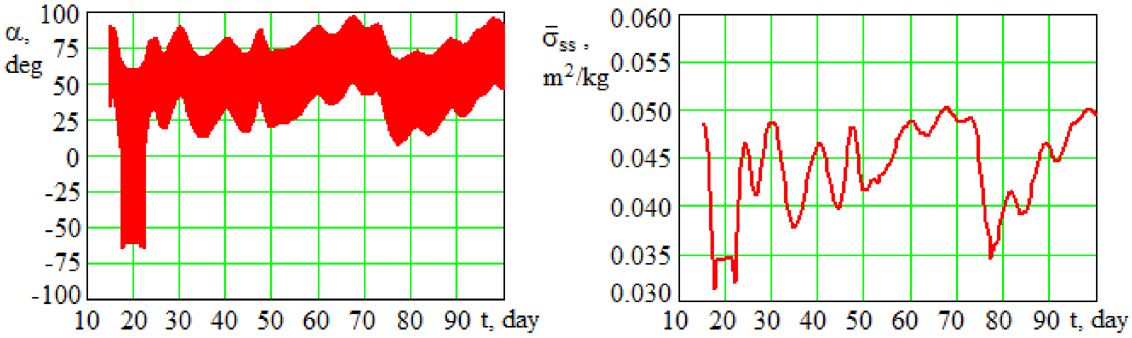


Fig.6. The change in the ratio of the maximum values of the aerodynamic and gravitational moments for the time period from 16 to 100 days

From the obtained results, it follows that the most likely movement relative to the SamSat-218D nanosatellite mass center is the transitional mode of movement between different equilibrium positions in the angle of attack. This mode of motion relative to the center of mass corresponds to the change in the angle of attack shown in Figure 7a and the change in the averaged ballistic coefficient over a specified period of time, shown in Figure 7b.



(a) (b)

Fig.7. (a) Change of angle of attack from 16 to 100 days;
(b) the change in the average ballistic coefficient from 16 to 100 days

At the beginning for a short time, the nanosatellite oscillates about the equilibrium positions of the attack angle α_* (about 65 degrees). Then, with an increase of the atmosphere density, the aerodynamic moment increases, and the nanosatellite motion transforms into oscillations with respect to the zero value of the angle of attack. Further, as the density of the atmosphere decreases, the aerodynamic moment decreases and the nanosatellite motion transforms into oscillations with respect to the changing equilibrium position α_* (in the range of 35-75 degrees). Such a complex nature of change of motion mode is manifested only at the observed comparable values of the aerodynamic and gravitational moments.

The lack of definiteness of the conclusion about the mode of motion is due to the observed practical immutability of the density of the atmosphere during the entire time period of motion of the nanosatellite, in view of the decrease in solar activity.

In the future, it is planned to continue monitoring the braking of the SamSat-218D nanosatellite in order to increase the reliability of conclusions about the variable dynamics of movement and fairness of the developed methodology for designing an aerodynamically stabilized nanosatellites.

5. CONCLUSIONS

The proposed approach to analyzing the angular motion of a nanosatellite using trajectory measurements, tested in the framework of the SamSat-218D “passive” experiment, allows the developers of nanosatellites performing an uncontrollable motion relative to the center of mass to make conclusions about the causes of the observed motion. If a nanosatellite after its launch was to change its configuration (for example, to open solar panels or pushrods), then the use of this approach allows concluding that the operation was successful or unsuccessful.

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