



Scientific and Technological Experiments on Automatic Space Vehicles and Small Satellites

## Mathematical modeling of radio tomographic ionospheres monitoring via satellite constellation

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### Abstract

The reconstruction of the electronic components distribution functions in Earth's ionospheric layer is an actual problem of applied space research. The reason is that the ionosphere as a medium of propagation significantly affects the operation of the various systems of navigation, location and communication. The mathematical modeling technique of the electron density spatial distribution reconstruction in Earth's ionosphere spherical layer using a constellation of small satellites was developed by the author. It assumes twenty - four satellites, that are arranged uniformly on the circular orbit, sound a ring layer of ionosphere with radio emission at frequencies of (150 and 400) MHz. It is shown that under certain conditions, the problem can be reduced to a few-view tomographic problem, and the desired reconstruction of the electron density distribution by fast convolution algorithms can be evaluated. The applied software package allowing to perform a full cycle of mathematical modeling both for two-dimensional and three-dimensional problems was developed. The author also developed specialized "few-views dynamic kernel" that provides an opportunity to obtain satisfactory reconstruction of wide range input projection data for this class of problems.

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### Nomenclature

$f_1, f_2$  coherent radio frequency in Hertz

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n	refractive index
P	pressure of dry air in Pascals
T	temperature in Kelvin
$P_w$	water vapor pressure in Pascals
$n_e$	electron density
f	carrier frequency in Hertz
$\Delta\phi$	phase difference in radians
TEC	total electron content in TEC Units
$\lambda$	wavelength of electromagnetic radiation in meters
$r_e$	electron radius in meters
N	electron concentration in TEC Units
dσ	space bin along the beam of sight satellite – satellite in meters
$a_i a_i$	chord
$L_i$	element of chord on circular area
$\Delta L_i$	element of chord on overlapping circular area
$g_i(r_i)$	orthogonal projection

## 1. Introduction

The state of an ionosphere defines the major vital processes on our planet. Currently, in all developed countries being monitored the ionosphere, in many different ways using different satellite constellations. Total electron content (TEC) is one of the most important characteristics of an ionosphere of Earth, however, today, on the territory of Russia its global monitoring is not carried out. Analytical models give a good estimate of this parameter on condition of a quiet geomagnetic situation, but in case of the perturbed ionosphere the estimate of TEC becomes significantly less exact [1] that negatively affects work various (in particular navigation) satellite systems.

Ionospheric monitoring is implemented now, as a rule, with the help of the ionospheric vertical sounding ground stations (IVSGS). IVSGS possible to determine the number of significant characteristics of the ionosphere with high precision, but they have several drawbacks: significant weight and size, high power consumption and high cost of maintenance. A promising approach to monitoring the ionosphere is to determine the main parameters of the ionosphere on the results of processing the received radio signals of global navigation satellite systems GLONASS and GPS. Radio sounding of the atmosphere using the signals of satellite navigation systems [2] and a net of ground stations is a way to ionospheric monitoring in real time, which is easily accessible and does not require large expenditures.

Despite all the advantages of modern monitoring systems, including the use GLONASS and GPS, these systems have one major drawback - the need to have a net of ground receiving stations. Methods and systems using ionospheric sounding via satellite constellations have not this deficiency[3,4]. In paper [3] proposed method radio tomographic ionospheric sounding using satellite constellation comprising satellites emitted coherent radio emission at frequencies  $f_1$ ,  $f_2$  and receiving satellites, detecting radiation of transmitters at these frequencies. Dual-frequency ionospheric sounding by radio signals is based on the existence of the phenomenon of dispersion of radio waves in the microwave range electron plasma forming the Earth's ionosphere. Refractive index of radio signal while passing through the atmosphere from a transmitter located on the emitting satellite is determined by the formula:

$$N_{ref} = (n - 1) \cdot 10^{-6} = 77.6 \frac{P}{T} + 3.37 \cdot 10^5 \frac{P_w}{T^2} - 40.3 \frac{n_e}{f^2}, \quad (1)$$

where P – pressure of dry air in Pascals,  $P_w$  – water vapor pressure in Pascals, T – temperature in Kelvin, f – carrier frequency in Hertz,  $n_e$  – electron density, n – refractive index.

## 2. Tomographic approach to processing satellite-satellite coherent radio signals

The total electron content (TEC) along the beam of sight from the antenna phase center of the receiver to the transmitter antenna is proportional to the difference between the phase incursions at two frequencies. Given that the phase velocity is equal in sign and opposite in value to the group velocity, it is easy to see that the TEC is proportional to the phase difference module or the difference between pseudoranges, determined from the navigation signals on two frequencies. However, not difficult to understand that the value of the TEC phase measurements can be determined only up to a fixed (within one session) constant.

The phase difference registration of electromagnetic radiation propagation through the ionosphere from transmitting satellites on receiving satellites, allows us to calculate the total electron content in the ionosphere on the propagation path of electromagnetic radiation in accordance with the following equation:

$$\text{TEC} \sim \Delta\phi = \lambda r_e \int_1^2 N d\sigma, \quad (2)$$

where TEC – a total electron content,  $\Delta\phi$  – the phase difference of coherent electromagnetic radiation after propagation through the ionosphere,  $\lambda$  – the wavelength of electromagnetic radiation with the lowest frequency  $f_1$ ,  $r_e$  – electron radius,  $N$  – electron concentration and  $d\sigma$  – space bin along the beam of sight satellite – satellite, having a dimension of length [5].

It should be noted also that the phase shift measurements by several orders more precisely than pseudoranges code measurements, so to determine the absolute TEC most convenient to use code and phase measurements together. The next step is to move from the measured absolute or relative delays along the inclined beam of sight to the vertical delays. Are two approaches to solving this problem:

- the first is that it is necessary to characterize all simultaneous measurements of only one average value of the vertical TEC values that are in the coordinate system of the receiver antenna;
- the second involves the calculation of "vertical" TEC values directly in the "underionospheric points" (points corresponding to the intersection of the beam of sight to the satellite with a hypothetical infinitely thin ionospheric layer located at a selected height).

The authors have developed a way of reconstruct the spatial distribution of the ionospheric electron density, which differs from the known methods, so that the use of simulation of the radio pulse propagation along the incident direction – distance between satellites, allows to convert the radio tomography problem to the usual problem of few-view reconstruction. This approach allows us to solve in the general case, the problem of direct 3D reconstruction of the TEC in an ionospheric spherical layer, or two-dimensional problem in its selected section.

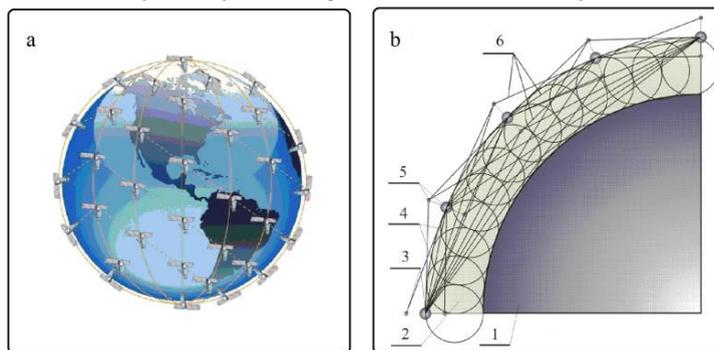


Fig. 1. (a) satellite constellation geometry; (b) one orbit plane satellite constellation of the arrangement for radio tomography ionospheric sounding, where 1 – Earth, 2 – ionosphere, 3, 5 – microsatellites, 4 – circular reconstruction zones, 6 – sounding beams.

The essence of this method is that the small satellites constellation arranged on several spatially located planar orbits deployed relative to one another, as shown in Fig. 1(a). With this configuration, the satellite constellations can solve the problem of three-dimensional TEC reconstruction. We explain the contents of the proposed TEC reconstruction method by the example of 2D reconstruction distribution function of the electronic component on the ring carrier according to some given section of the ionosphere. To do this, necessary to put 24 minisatellite into a flat circular orbit, as shown by simulation experiments. Each minisatellite must contain receivers and transmitters operating on frequencies  $f_1$ ,  $f_2$ . Moreover, for registering the phase components for each frequency, each minisatellite in its composition must have two nanosatellites provided with receivers for these frequencies. After separation from the microsatellites delivery device and their distribution along a circular orbit at equal distances from each other, each of them produces a nanosatellites "start" in the radial direction as shown in Fig. 1(b). Nanosatellites moved from its carrier at distances  $(2 \div 5)$  km, and remain fixed at these distances. The nanosatellites retention provided with the microcable locking systems. This satellite constellation configuration enables along this sound direction - 6 (Fig. 1(b)) register phase components of the sound pulse, the same way as it is done ground antenna devices detect radiation by navigation satellite constellation. In addition, each minisatellite contains a transceiver (ratings frequencies: 150, 400 MHz), it must have orientation engines (ion-plasmas type), laser rangefinder, gyroscopic device, etc. In each minisatellite is also necessary to place the pre multiprocessor module processing raw data, and radio channels for communication between satellites and orbital or ground system support.

Inverse problem of the TEC reconstruction on the ring carrier can be reduced to the usual few-view reconstruction problem based on the convolution method with few-view kernel and back-projection procedure in two ways. The first is that it is possible to reformulate Radon Theorem for ring carrier, provided that the desired functions can be predetermine on the basis of experimental data on the circular area of reconstruction. However, due to the fact that the height of the ionosphere is taken equal to 1,500 km, and the radius of our planet 6300 km, ring reconstruction zone is too small, in the sense that the amount actually received chord data is negligible compared with their number on a circular carrier radius 7800 km. Furthermore, it should be noted that chord data that must be subsequently result in the projection data, can be determined in a rather small angle of convergence. Taking into account, that the spatial resolution, even in the two-dimensional case, must determine the unit area of not less than  $(500 \div 500) \text{ m}^2$ , i.e. each projection must contain 15600 counts and reconstructed format respectively  $(15600 \div 15600)$  elements, it becomes clear that this approach will require enormous computational cost, making it ineffective.

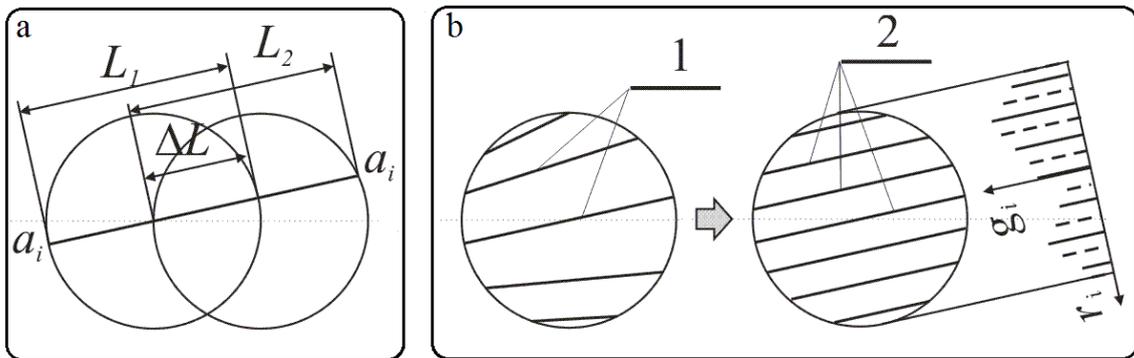


Fig. 2. (a) overlapping circular zones; (b) projection geometry.

Another way proposed by the authors is that, for example, for the 2D reconstruction problems:

- The ring layer of the ionosphere is represented as a set of circular (elementary) carriers whose diameter is equal to the height of the ionosphere, overlapping, see Fig. 1(b).
- Thus, the chord element between satellites emitting and receiving radio-frequency pulse can be represented by the sum of segments  $a_i$  (see Fig. 2(a)).

- Each segment  $a_i$ , in turn, is determined by chords of elementary circular reconstruction areas  $L_1, L_2$ , with an area of overlap  $\Delta L$ .
- If the parameters are known a priori, such as the ionosphere air pressure, steam, temperature, etc. see equation (1), we can calculate the attenuation by modeling the radio emission at each frequency for chord directions  $L_{1i}, L_{2i}$ , taking into account the areas of overlap  $\Delta L_i$ .
- Therefore, using data for the absorption in the directions of the probing beams – 6 (Fig. 1(b)) can create a set of chord data – 1, for each unit of the circular area reconstruction (Fig. 2 (b)).
- Next step is recalculate chord data from fan geometry to orthogonal (Fig. 2(b)), predetermine to the specified reconstruction format, i.e. formation of classical orthogonal projection  $g_i(r_i) - 2$  (Fig. 2 (b)).

However, as in the first case, with this approach, we can get a set of projection data in a rather small angle of convergence by the order of  $(15 \div 30)^\circ$ . This is true for minisatellites adjacent located in zones beam of sight with respect to each other (Fig. 1(b)). To do efficient reconstruction procedure using fast convolution algorithms and operations of back projection must be predetermined projection data in Fourier space, using the symmetry properties of the Fourier images and a priori data about air pressure, water vapor etc. The calculation of the required number of intermediate projections made on circular harmonics in polar coordinates of Fourier space. Further, being transformed one-dimensional Fourier images calculated wanting projections in signal space, and convolving projection functions with of few-view low-frequency kernel. After the operation of back projection and transform data in a Cartesian coordinate system, we obtain the desired reconstruction of elementary circular carrier - according to the geometry shown in Fig. 1(b) of the 48 elementary carriers for ionospheric ring. Given additivity overlay zones crossing performed the final reconstruction of the desired distribution of the TEC on the ring area of the ionosphere.

### 3. Methods verification

In the diagnosis of physical medium by solving the inverse problem (including the radio tomography of the ionosphere) is usually not possible to verify the data obtained by direct methods in full, that is, to get the same amount of information by probing methods or other methods. The accuracy of the reconstruction parameters of the medium in these cases is determined on the basis of numerical modeling. To assess the quality of ionospheric parameters reconstruction consistently solve direct and inverse problems. The direct problem is to obtain the integral characteristics of the medium, in this case, the TEC, for a given distribution of the electron density and the available set of pathways sounding signals in this area (see. Fig. 2(a) and (b)). Rate attenuation the radio emission in these directions, it is possible using known empirical model of the ionosphere.

Some common sources of electron content are [6]:

- Global TEC maps computed by the International GNSS Service (IGS) from a global network of dual-frequency receivers;
- Regional TEC models, which provide better accuracy, by means of a better temporal and spatial resolution, thanks to the availability of a dense permanent receivers network (e.g. for Japan, Europe or USA);
- Predicted TEC models used by GNSS (Klobuchar model for GPS or NeQuick for the future Galileo system [7]);
- Empirical standard ionospheric models, which are based on all available data sources such as the International Reference Ionosphere – IRI (Bilitza and Reinisch 2007) or Parameterized Ionospheric Model – PIM (Daniell et al. 1995) [8].

It should be noted NeQuick model. NeQuick is a semi-empirical model that describes spatial and temporal variations of the ionospheric electron density. It is based on the Di Giovanni Radicella (DGR) ionospheric profiler (Di Giovanni and Radicella 1990) and provides both vertical or slant electron content for any specified path (Hohegger et al. 2000, Radicella and Leitinger 2001). NeQuick model uses the peaks of the three main ionospheric regions (E, F1, and F2) as anchor points (Radicella and Leitinger 2001; Leitinger et al. 2005). The electron density at any location is computed starting from the characteristic parameters such as peak electron density and peak height; STEC is computed by integrating TEC along the signal path. NeQuick model is at basis of the real-time

ionospheric correction model algorithm used for Galileo single-frequency positioning (Radicella and Leitinger 2001, Schluter et al. 2004, Nava et al. 2005).

Also among the empirical models of the ionosphere most widespread International Reference Ionosphere (IRI) is an international project sponsored by the Committee on Space Research (COSPAR) and the International Union of Radio Science (URSI). For given location, time and date, IRI describes the electron density, electron temperature, ion temperature, and ion composition in the altitude range from about 50 km to about 2000 km; and also the electron content. It provides monthly averages in the non-auroral ionosphere for magnetically quiet conditions. The major data sources are the worldwide network of ionosondes, the powerful incoherent scatter radars, the ISIS and Alouette topside sounders, and in situ instruments on several satellites and rockets.

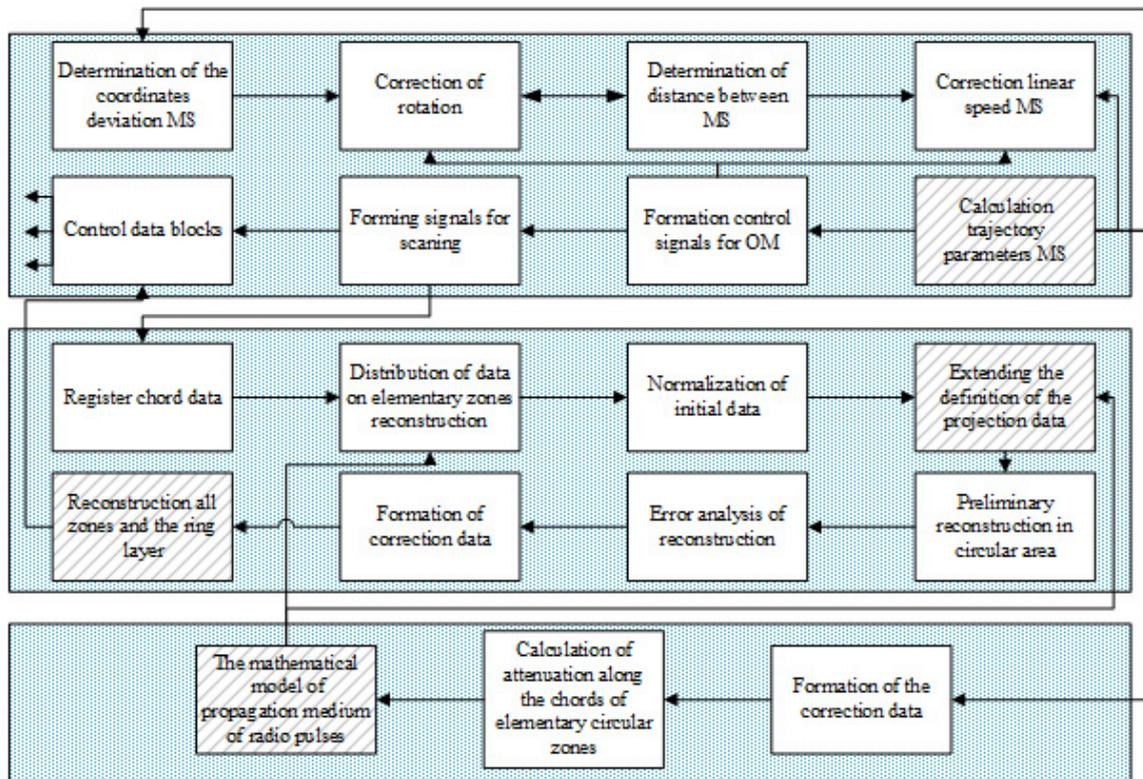


Fig. 3. Block diagram of the application package for the reconstruction of the TEC on the ring carrier.

#### 4. Methods realization

Fig. 3 shows a block diagram of the application package, which allows simulating the two-dimensional reconstruction procedure of the TEC distribution functions in the ionospheric ring zone. For convenience, package program modules presented in three segments. The first segment contains modules that are responsible for the location of small satellites into the orbit, their mutual orientation, and modules forming control signals to optimize the location of each MS into the orbit, using orientation microengine (OM). The second segment shows the modules

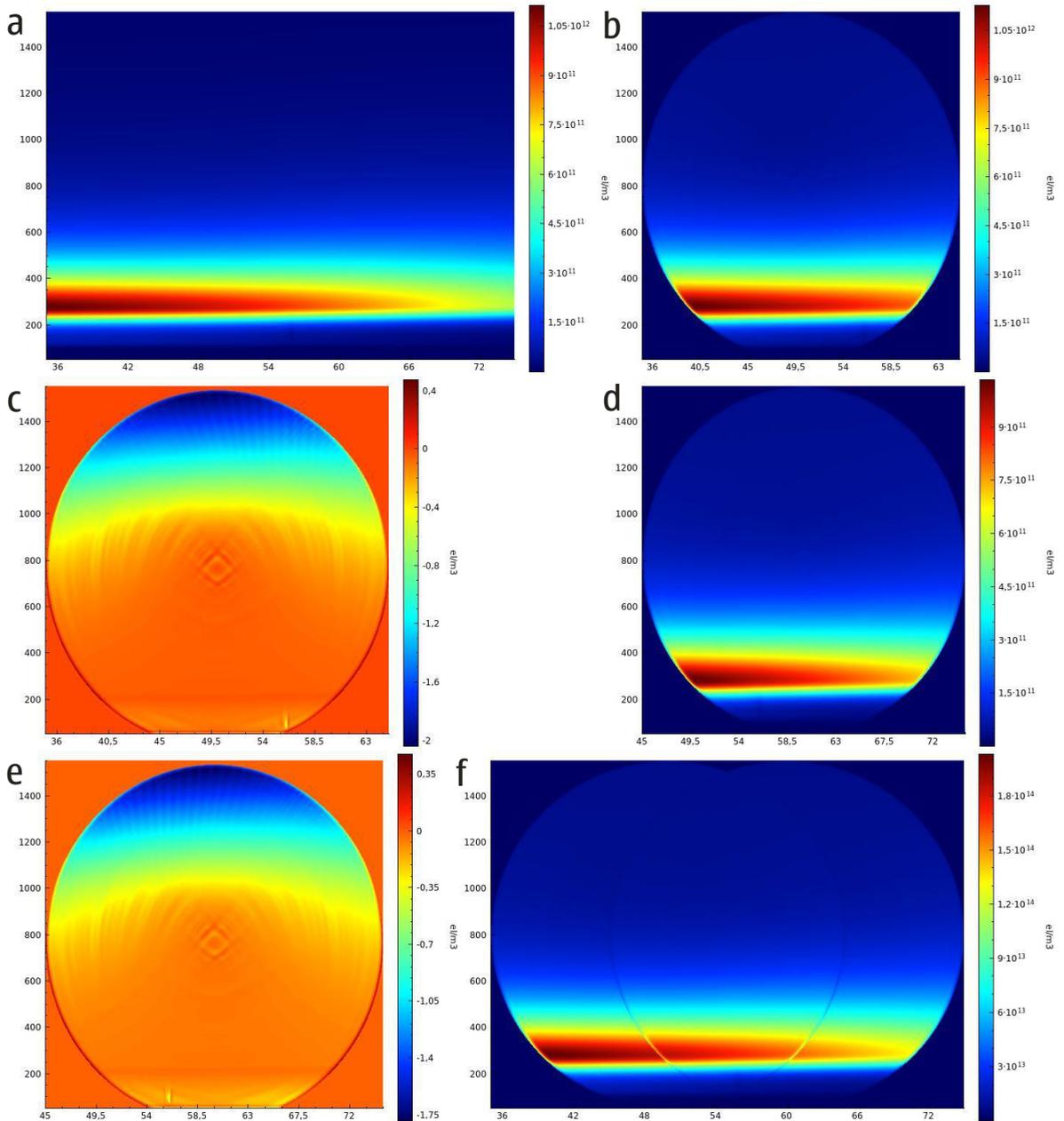


Fig. 4. (a) NeQuick model TEC distribution function (lat. = 53° deg. N); (b), (d) reconstruction of the desired TEC distribution functions on circles (the number of projections is equal 180), taken from (a); (c), (e) relative error fields corresponding to (c), (e); (f) distribution function on two overlapping circles without postprocessing.

responsible for the reconstruction of the desired TEC distribution in the ring section. The third segment includes modules associated with process modeling attenuation of radio emission along each sound chord in elementary circular areas of reconstruction. This takes into account the functional distribution of air pressure, the partial pressure of water vapor with height. The grayed marked out modules, starting in its class. It is assumed that the considered control procedures, simulation and reconstruction can be performed multiprocessor computing system

installed on board each MS.

Fig. 4(a) shows the model TEC distribution function according to [2], obtained by tomography of the ionosphere with the help of navigation satellites and a net of ground receiving stations. Fig. 4(f) is an example of reconstruction of the desired TEC distribution using the methods described above. Fig. 2(b), (d) shows a display example of the required distribution functions as a projection image and Fig. (c), (e) shows their relative error fields. In a model experiment reconstruction format has been selected ( $512 \times 512$ ) of the elements at 512 gradations in amplitude. Choosing this format is associated with time constraints standard quad-core processor PC. On multiprocessor systems, procedures implementing distributed computing environment in UNIX format, the reconstruction can be increased by an order that will make it possible to get the resolution ( $500 \times 500$ ) m<sup>2</sup> and above.

## 5. Conclusion

In this article, we studied the possibility of reconstruct the vertical distribution of ionospheric tomography method without the use of ground receiving stations. To achieve the required accuracies are sufficient arranged on satellite constellation planar orbit in the amount of 24 satellites. The proposed variant of the satellite constellation provides a sufficient amount of projection data for the tomography application to considered at the circular zones from ionospheric ring. Reducing the number of satellites can be achieved through improvements applied methods of tomography to this problem. Such an approach to the determination of the vertical distribution of the electron density in the ionosphere can be used in the future when you create a space system for monitoring geophysical conditions.

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