Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 49-17 (2016) 206–211

Technology for determining the local vertical of nanosatellite by processing videoimages of the Earth horizon

Igor Lomaka*. Igor Belokonov. Efim Ustugov*****

**Samara State Aerospace University*
Russia (e-mail: igorlomaka63@gmail.com).
***Samara State Aerospace University*
Russia (e-mail: ibelokonov@mail.ru).
****Samara State Aerospace University*
Russia (e-mail: efim163@gmail.com).

Abstract: This paper describes the algorithm for determining the local vertical for nanosatellite, based on the sight of the horizon of the Earth in the visible wavelength range. The application of this algorithm will allow determining the direction of the local vertical to within 1 degree. The results of numerical experiments show that the proposed algorithm works in a wide range of orientation angles. The article also examines the semi-natural experiment to determine the pitch angle by processing the video coming from the camera. The experiment showed that the horizon sensor based on this algorithm would have an accuracy less than 1 degree in the area of ± 20 degrees pitch and ± 20 degrees yaw.

***Keywords**:* nanosatellite, video navigation, the algorithm, image processing, attitude determination.

1. INTRODUCTION

Every year the number of nanosatellites is constantly increasing, so expands and the range of problems they can solve. One of this problems is the remote sensing of the earth. This problem requires the maintenance of a high-precision orientation of sensitive equipment relative to the local vertical. For example for a 500 km altitude orbit, 0.51 attitude control accuracy translates into approximately 4.5 km of spatial uncertainty on ground (Selva, D(2012)). Typically, this class of spacecraft has on-board inertial sensors that have integral error, so in solving problems of remote sensing, the role of non-inertial sensors, such as the horizon sensor, a star sensor, sun sensor is increasing. This article covers the creation of algorithm based on image processing of the Earth horizon. The purpose of the algorithm is to provide a direction vector of the local vertical by the analysis of two contemporary obtained images of Earth horizon.

The proposed algorithm is different from the known that the calculated vector is invariant to the horizontal displacement of the image and nanosatellite rotation around the longitudinal axis, and also does not depend on focal distance of the camera.

In the second part is a brief description of the algorithm, as well as technical systems on which it can be implemented.

In the third part describes the mathematical modeling of the algorithm and key results.

In the fourth part describes the conduct of the experiment and compare the data obtained from the simulation data.2.

2. ALGORITHM OF VIDEONAVIGATION AND HORIZON SENSOR DESIGN

2.1 Algorithm of videonavigation

The algorithm for determining the direction of the local vertical is as follows:

• The image obtained from the camera stands out the horizon line, which is a circular arc;

• Determine the center and the radius of the arc of a circle;

• Construct the line connecting the center of the resulting arc to the center of the frame;

• To construct the normal line;

• The same procedure is performed and the second camera;

• The cross product gives the desired normal vector direction of the local vertical in the associated coordinate system;

It should be noted that the line connecting the center of the arc resulting from the center of the frame is the trace of the plane that passes through the Earth's center, center of mass of the satellite and perpendicular to the plane of the camera frame. Thus the intersection of two such planes gives a line passing through the center of the Earth and the center of mass of the satellite, that is, the local vertical.

2405-8963 © 2016, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.
Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.09.036

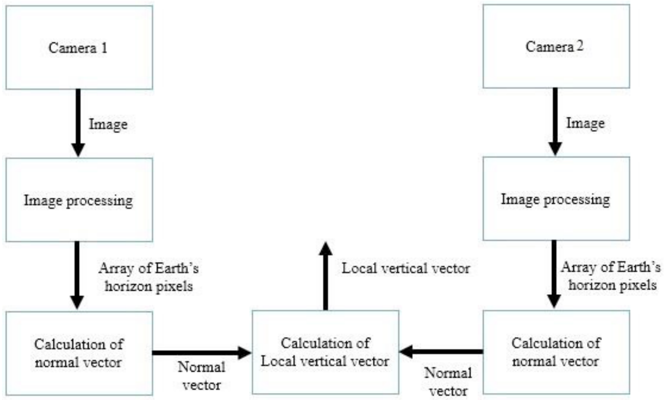

Fig. 1. Algorithm scheme

The input data for the calculation of the normal vector is an array of pixels, describing the border Earth - space, obtained by processing the image by Canny edge detector. At first arc selected three points - two outer points $P_1(x_1, y_1)$, $P_3(x_3, y_3)$, and one inner $P_2(x_2, y_2)$. Then the coordinates of the arc center are determined (x_c, y_c) and radius R_c (fig. 2)

$$m_a = y_2 - \frac{y_1}{(x_2 - x_1)}, \tag{1}$$

$$m_b = y_3 - \frac{y_2}{(x_3 - x_2)}, \tag{2}$$

$$x_c = \frac{m_a \cdot m_b \cdot (y_1 - y_2) + m_b \cdot (x_1 + x_2) - m_a \cdot (x_2 - x_1)}{2 \cdot (m_a - m_b)}, \tag{3}$$

$$y_c = -\frac{1}{m_a} \cdot \left(x_c - \frac{x_1 - x_2}{2}\right) + \frac{y_1 - y_2}{2}, \tag{4}$$

$$R_c = \sqrt{(x_c + x_2)^2 + (y_c + y_2)^2}, \tag{5}.$$

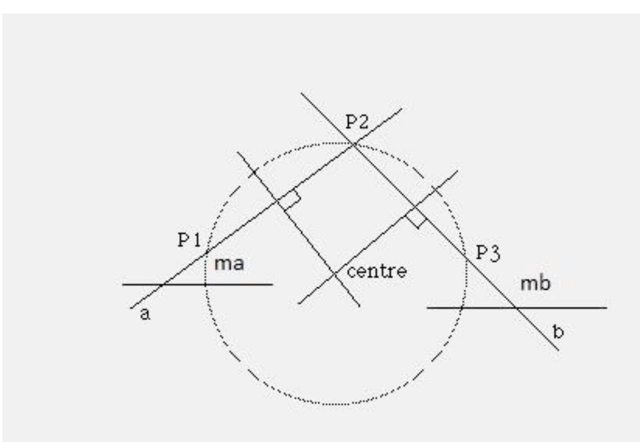

Fig. 2. Determination of the center.

After that, you need to connect the center of the arc to the center of the frame with a line, which is described by the equation:

$$y = kx + m \tag{6}$$

Where *k* and *m* coefficients that are calculated according to the formulas:

$$k = \frac{y_k - y_c}{x_k - x_c} \tag{7}$$

$$m = y_c - \frac{y_k - y_c}{x_k - x_c} \cdot x_c \tag{8}$$

Transform equation (6) to the form $A(x - x_0) + B(y - y_0) = 0$, where A and B - of the vector perpendicular to a given line that you want to find. To find A and B equate $x_0 = x_c$, $y_0 = y_c$ and form a system of linear equations

$$\begin{cases} A(x_1 - x_c) + B(y_1 - y_c) = 0 \\ A(x_2 - x_c) + B(y_2 - y_0) = 0 \end{cases} \tag{9}$$

where (x_1, y_1) и (x_2, y_2) - the coordinates of two random points, calculated according to the formula 6).

2.2 Horizon sensor design

Prospective embodiment of the system is shown on Figure 3. It is a device consisting of a photodetector and a circuit board with onboard computing device. The photodetectors are positioned so that their sight axis perpendicular to each other and lie in a plane orthogonal to the longitudinal axis of the nanosatellite to acquire images in the associated with the satellite coordinate system.

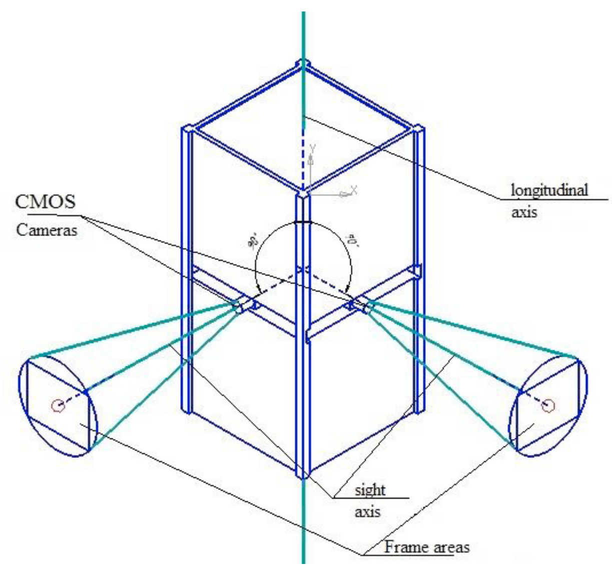

Fig. 3. Horizon sensor design.

3. SIMULATION OF THE ALGORITHM

The nanosatellite is located at orbit with altitude H above the Earth. Accepted assumption that the Earth is a sphere, described by the equation:

$$x^2 + y^2 + z^2 = R^2 \tag{10}$$

At the nanosatellite are located two identical photodetectors which receive two simultaneous snapshots of the Earth horizon. Axis sight unit vector is describing photodetectors, and (in the geocentric coordinate system), respectively. Snapshots have a resolution $n \times m$ of pixels.

The projection of the Earth's horizon to the plane of the photodetector is an arc of a circle, which is described by parametric equations:

$$\begin{cases} X_o = C_x + \frac{R_{\text{сеч}}}{\sqrt{n_{kx}{}^2+n_{kz}{}^2}} \cdot \left(n_{kz} \cdot \cos(t) - \frac{n_{kx} \cdot n_{ky} \cdot \sin(t)}{\sqrt{n_{kx}{}^2+n_{ky}{}^2+n_{kz}{}^2}}\right) \\ Y_o = C_y + \frac{R_{\text{сеч}}\sqrt{n_{kx}{}^2+n_{kz}{}^2}}{\sqrt{n_{kx}{}^2+n_{ky}{}^2+n_{kz}{}^2}} \cdot \sin(t) \\ Z_o = C_z - \frac{R_{\text{сеч}}}{\sqrt{n_{kx}{}^2+n_{kz}{}^2}} \cdot \left(n_{kx} \cdot \cos(t) + \frac{n_{kz} \cdot n_{ky} \cdot \sin(t)}{\sqrt{n_{kx}{}^2+n_{ky}{}^2+n_{kz}{}^2}}\right) \end{cases} \quad (10)$$

where $t \in [0;2\pi)$.

Viewing angle of the cone of photodetectors is known. It is necessary to determine the vector of the local vertical in the associated coordinate system and determine the deviation of the longitudinal axis from the local vertical.

In the following assumptions were made:

The Earth is a sphere;

Axis of sight photodetectors pass through the center of mass of nano-satellite;

Influence of the atmosphere on the image distortion is not considered.

To test the algorithm, a mathematical model must associate the height of the satellite's orbit and its orientation with images that get cameras mounted on its board.

Field of view of the camera is a pyramid, inscribed in the cone of the camera. The angle of the cone is α. The base of the pyramid is perpendicular to the axis of the camera sight. The unit vector $\vec{n}_{1k}$ describes the axis of sight of the camera (Figure 3). The faces of the pyramid are determined by vectors $\overrightarrow{n_1}$, $\overrightarrow{n_2}$, $\overrightarrow{n_3}$, $\overrightarrow{n_4}$.

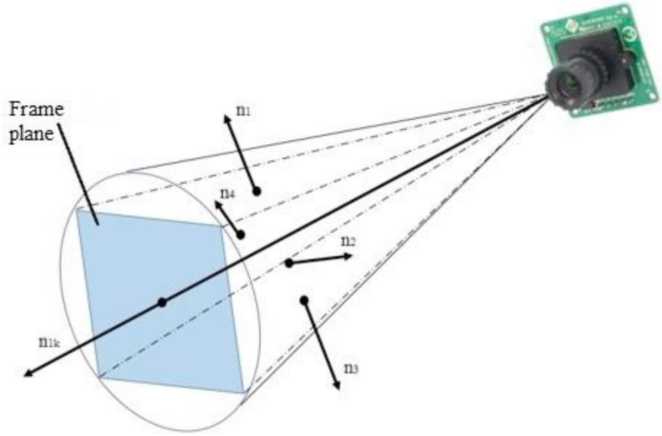

Fig. 3. Field of view.

Place the camera in the point A on the Earth's surface (Figure 4) and find the point of the horizon of the earth, which are projected onto the photodetector sensor array, that is, we get the image of the horizon of the Earth at a given height and orientation of the camera.

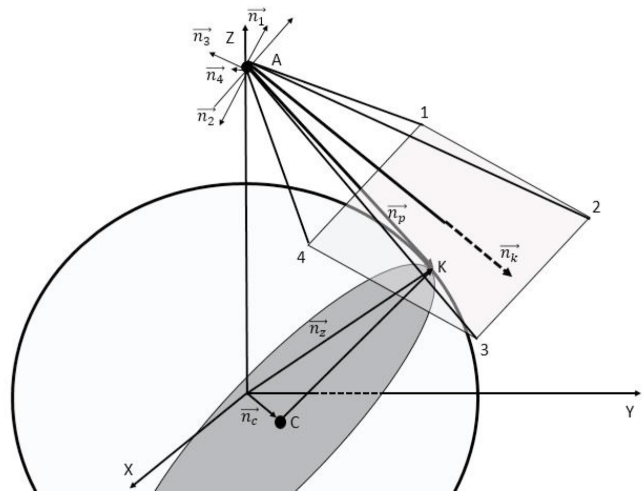

Fig. 4. Camera position.

Let find the coordinates of the point of tangency K as the intersection of the three planes, that uniquely identifies a point in space. The equation of the first plane is

$$v_x(K_x - 0) + v_y(K_y - 0) + v_z(K_Z - 0) = 0 \quad (11)$$

as it passes through the point $(0\ 0\ 0)$ perpendicular to the vector $\vec{v}$, where $\vec{v} = \vec{n}_z \times \vec{n}_p$. The equation of the second plane is

$$n_{Zx}K_x + n_{Zy}K_y + n_{Zz}K_Z = (R + H)n_{Zz} \quad (12)$$

as it passes through the point $\left(0\ 0\ (R + H)\right)$ perpendicular to the vector $\overrightarrow{n_z}$. The equation of the third plane is

$$n_{kx}K_x + n_{ky}K_y + n_{kz}K_Z = 0 \quad (13)$$

as it passes through the point $(0\ 0\ 0)$ perpendicular to the vector $\overrightarrow{n_k}$. So the coordinates of the point of tangency K are determined by solving a system of linear equations

$$\begin{cases} v_xK_x + v_yK_y + v_zK_Z = 0 \\ n_{Zx}K_x + n_{Zy}K_y + n_{Zz}K_Z = (R + H)n_{Zz} \\ n_{kx}K_x + n_{ky}K_y + n_{kz}K_Z = 0 \end{cases} \quad (14)$$

For brevity, we write $N \cdot P = D$ where

$$N = \begin{pmatrix} v_x & v_y & v_z \\ n_{Zx} & n_{Zy} & n_{Zz} \\ n_{kx} & n_{ky} & n_{kz} \end{pmatrix}, P = \begin{pmatrix} K_x \\ K_y \\ K_z \end{pmatrix} \text{ and } D = \begin{pmatrix} 0 \\ (R + H)n_{Zz} \\ 0 \end{pmatrix}$$

For coordinates of section center point C we will write:

$$N = \begin{pmatrix} v_x & v_y & v_z \\ n_{cx} & n_{cy} & n_{cz} \\ n_{kx} & n_{ky} & n_{kz} \end{pmatrix}, P = \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} D = \begin{pmatrix} 0 \\ 0 \\ n_{kx}K_x + n_{ky}K_y + n_{kz}K_Z \end{pmatrix}$$

For the coordinates of outer points of the frame 1,2,3 and 4, respectively, we will write

$$N = \begin{pmatrix} n_{1x} & n_{1y} & n_{1z} \\ n_{3x} & n_{3y} & n_{3z} \\ n_{kx} & n_{ky} & n_{kz} \end{pmatrix}, P = \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} D = \begin{pmatrix} n_{1z}(R + H) \\ n_{3z}(R + H) \\ n_{kx}C_x + n_{ky}C_y + n_{kz}C_Z \end{pmatrix}$$

$$N = \begin{pmatrix} n_{1x} & n_{1y} & n_{1z} \\ n_{4x} & n_{4y} & n_{4z} \\ n_{kx} & n_{ky} & n_{kz} \end{pmatrix}, P = \begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix} D = \begin{pmatrix} n_{1z}(R + H) \\ n_{4z}(R + H) \\ n_{kx}C_x + n_{ky}C_y + n_{kz}C_Z \end{pmatrix}$$

$$N = \begin{pmatrix} n_{2x} & n_{2y} & n_{2z} \\ n_{4x} & n_{4y} & n_{4z} \\ n_{kx} & n_{ky} & n_{kz} \end{pmatrix}, P = \begin{pmatrix} X_3 \\ Y_3 \\ Z_3 \end{pmatrix} D = \begin{pmatrix} n_{2z}(R + H) \\ n_{4z}(R + H) \\ n_{kx}C_x + n_{ky}C_y + n_{kz}C_Z \end{pmatrix}$$

$$N = \begin{pmatrix} n_{2x} & n_{2y} & n_{2z} \\ n_{3x} & n_{3y} & n_{3z} \\ n_{kx} & n_{ky} & n_{kz} \end{pmatrix}, P = \begin{pmatrix} X_4 \\ Y_4 \\ Z_4 \end{pmatrix} D = \begin{pmatrix} n_{2z}(R + H) \\ n_{3z}(R + H) \\ n_{kx}C_x + n_{ky}C_y + n_{kz}C_Z \end{pmatrix}$$

After you need to define the center section of the formula as a frame coordinates found

$$R_s = \sqrt{(K_x - C_x)^2 + (K_y - C_y)^2 + (K_z - C_z)^2}$$

Now, knowing the coordinates of points of the circle and the coordinates of the extreme points of the frame define the points of the circle f which belong to the rectangle of the frame. If the point is in the frame, then the camera sees it.Define hit any point of the circle with the coordinates T (T_x, T_y, T_z) in a rectangle with vertices 1,2,3,4 (Figure 5) (coordinates of the vertices are known).Construct a vector from the point B in the top of the rectangle. The point B belongs to the rectangle, if the sum of the areas of triangles T12, T23, T34, and T41 is a rectangle area in 1234.

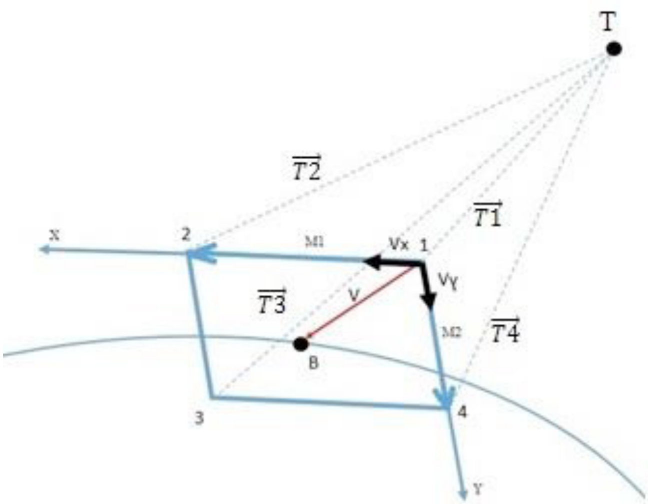

Fig. 5. Camera position.

The areas of triangles determined by formulas (15) - (18), and the area of the rectangle by the formula (19).

$$S_{T12} = 0{,}5 \cdot \left|\overrightarrow{T1} \times \overrightarrow{T2}\right| \quad (15)$$

$$S_{T23} = 0{,}5 \cdot \left|\overrightarrow{T2} \times \overrightarrow{T3}\right| \quad (16)$$

$$S_{T34} = 0{,}5 \cdot \left|\overrightarrow{T3} \times \overrightarrow{T4}\right| \quad (17)$$

$$S_{T41} = 0{,}5 \cdot \left|\overrightarrow{T4} \times \overrightarrow{T1}\right| \quad (18)$$

$$S_{1234} = \left|\overrightarrow{32} \times \overrightarrow{34}\right| \quad (19)$$

Thus we investigated all points of the circle belonging to the frame. It is needed to move from a geocentric coordinate system in the coordinate system of the photodetector. Consider the conversion of the coordinates of an arbitrary point B of the geocentric system of coordinates in the detector coordinate system (Figure 5).

X-axis passes through the points 1 and 2, and the Y axis at the point 1 and 4. To recalculate the coordinates, it is necessary to select a unit basis. To do this, choose two orthogonal vectors $\vec{M}_1$ and $\vec{M}_2$. Vector $\vec{M}_1$ has coordinates $(X_2 - X_1,\ Y_2 - Y_1, Z_2 - Z_1)$, vector $\vec{M}_2$ – $(X_4 - X_1,\ Y_4 - Y_1, Z_4 - Z_1)$. Transform vectors $\vec{M}_1$ and $\vec{M}_2$. To unit vectors and denote respectively $\vec{V}_x$ and $\vec{V}_y$. Vectors $\vec{V}_x$ and $\vec{V}_y$ will have coordinates $\left(\frac{M_{1x}}{|M_1|}, \frac{M_{1y}}{|M_1|}, \frac{M_{1z}}{|M_1|}\right)$ and $\left(\frac{M_{2x}}{|M_2|}, \frac{M_{2y}}{|M_2|}, \frac{M_{2z}}{|M_2|}\right)$. Since the matrix has the dimension of the photodetector $(m \times n)$ pixels, we introduce scaling factors for each axis $p_x = \frac{m}{|M_1|}$ and $p_y = \frac{n}{|M_2|}$. Consider a random point of the arc B $(B_x,\ B_y,\ B_z)$, construct vector $\vec{V}(B_y - X_1,\ B_y - Y_1, B_z - Z_1)$. To get coordinates of B point in the coordinate system of the photodetector expand vector $\vec{V}$ on basis $\vec{V}_x\ \vec{V}_y$, then multiply the corresponding projections on the scaling factor and rounded to the nearest whole

$$B_x^f = \left(V_x V_{xx} + V_y V_{xy} + V_z V_{xz}\right) px \quad (20)$$

$$B_y^f = \left(V_x V_{yx} + V_y V_{yy} + V_z V_{yz}\right) py \quad (21)$$

A similar transformation is applied to all points of the horizon that were caught in the frame. An example of simulation is show on Figure 6.

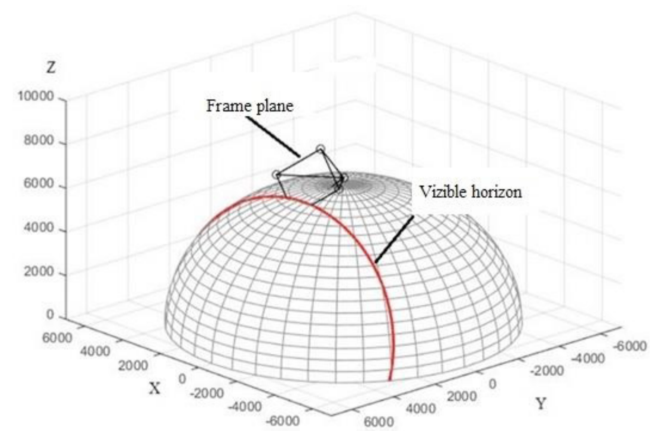

Fig. 6. An example of simulation.

Using the resulting model to determine the range of applicability of the pitch and yaw system with the parameters given in Table 1.

Angle of view	56 deg
Orbit height	300 km
Sensitive array	640x480 pixels

Table 1 System parametrs.

The area of applicability is calculated by statistical tests as follows: generates random angles of pitch and yaw in a range from -90 to 90 degrees, and checked visibility of Earth's horizon by two photodetectors, if both photodetectors see the horizon, the point is added to the corresponding coordinates on the plane, if at least one not see the horizon, the point is not added. At each point is calculated error in determining the direction of the local vertical. The area of applicability is presented in Figure 7 (held 30,000 iterations).

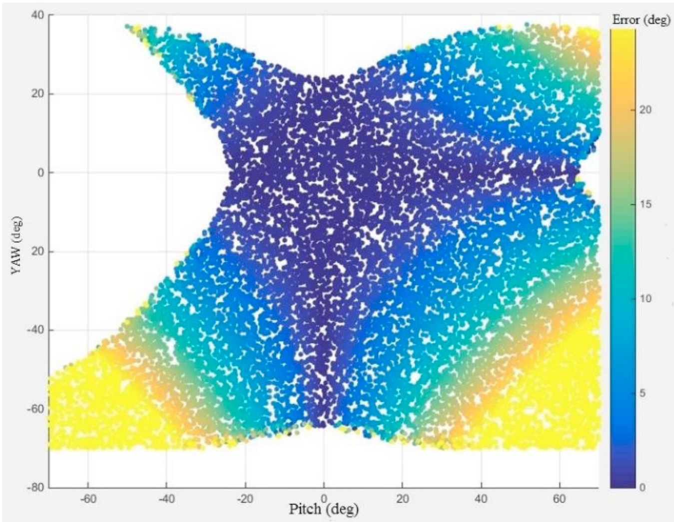

Fig. 7. The area of applicability.

According to the results of modeling applicability is a figure lying in the range from -70 to 70 degrees and a pitch in the range of -70 to 40 degrees yaw. It can be concluded that in the ± (20 - 22) degrees pitch and yaw algorithm has an error less than 1 degree. Such precision is required, such as the orientation of nanosatellites for remote sensing, and in some cases may even be redundant (Shakhmatov(2015)). Thus, we can assume successful verification algorithm.Using the resulting model to investigate the range of applicability for systems with parameters shown in Table 2 at different heights.

Angle of view	Orbit altitude	Sensitive array
45 deg	100 – 1050 km	640x480 pixels
50 deg	100 – 1050 km	640x480 pixels
55 deg	100 – 1050 km	640x480 pixels
60 deg	100 – 1050 km	640x480 pixels

Table 2 Systems parameters.

A feature of these systems is in the area with an error of less than 1 degree, which is kept at a constant angle camera solution to a certain critical height (H_{cr}). We will consider this area the most preferred for the functioning of the system. We define this area of a circle of radius R = 20 degrees. Further areas of applicability repeat the simulation and we will count the number of points that belong to this area. The ratio of the number of points that fall in the region (N_o), the total amount of N points shows the share of the points got in the preferred area of the orientation angles (Figure 8) $\frac{N_o}{N}$.

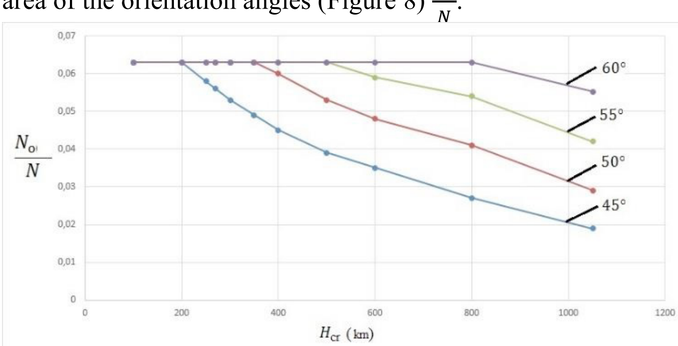

Fig. 8. Dependence $\frac{N_o}{N}$ on the altitude.

The constancy of the relation $\frac{N_o}{N}$ indicates immutability of preferred area of use of the system, reducing the field suggests contraction (Figure 10). Since Most of nano-satellites are launched into orbit with a height of 200 - 400 km, the best camera angle of the solution must lie in the range of 45 to 53 degrees.

4. EXPERIMENT

To test the algorithm was carried out semi-natural experiment to determine the pitch angle. The experimental unit is shown in Figure 11.

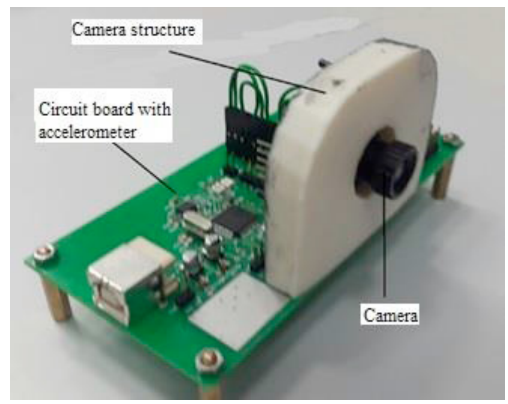

Fig. 11. Experimental unit.

In front of the camera was located test image with the earth's horizon. The experimental setup was deflected by a certain angle of pitch, which was calculated on the accelerometer at the same time made a video recording. Experimental data presented in Figure 12 and Figure 13.

The standard deviation of the vertical video was 0.752 degrees. The standard deviation of the model was 0.038 degrees. The difference between the model and experimental values due to the presence of noise on the snapshots and imperfect experimental setup.

5. CONCLUSION

The developed algorithm has an error of less than 1 degree in a certain range of angles of orientation. The validity of the algorithm was confirmed by semi-natural experiment. The disadvantage of this algorithm is the inability to determine the orientation of the shadow portion of the orbit. The algorithm can be used for orientation of nano-satellite equipment for local vertical

REFERENCES

Selva, D., & Krejci, D. (2012). A survey and assessment of the capabilities of cubesats for earth observation. *Acta Astronautica, 74*, 50-68. doi:10.1016/j.actaastro.2011.12.014

Shakhmatov, E., Belokonov, I., Timbai, I., Ustiugov, E., Nikitin, A., & Shafran, S. (2015). SSAU project of the nanosatellite SamSat-QB50 for monitoring the earth's thermosphere parameters. Paper presented at the *Procedia Engineering, , 104* 139-146. doi:10.1016/j.proeng.2015.04.105

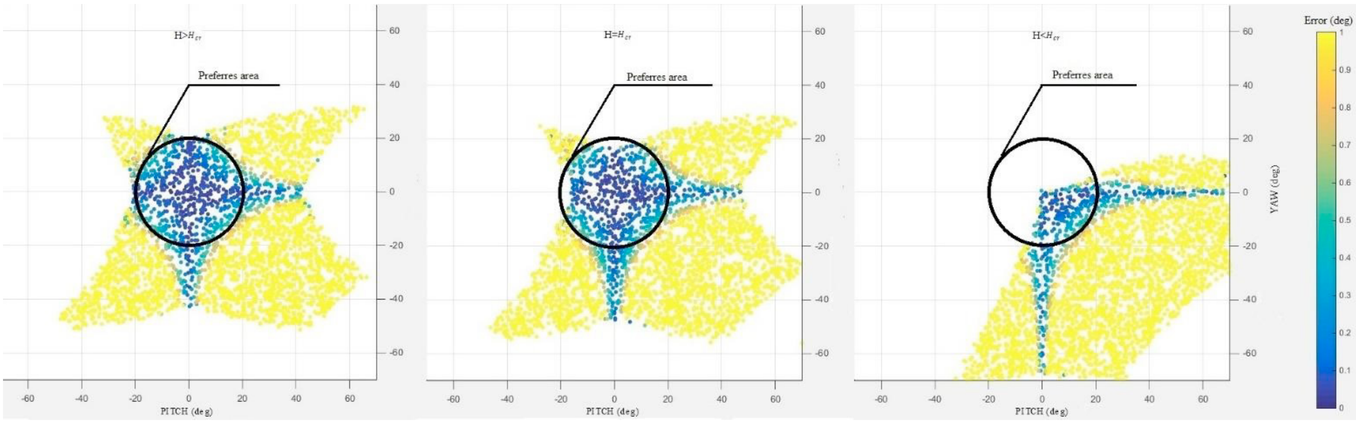

Fig. 10. Results of numeric experiment.

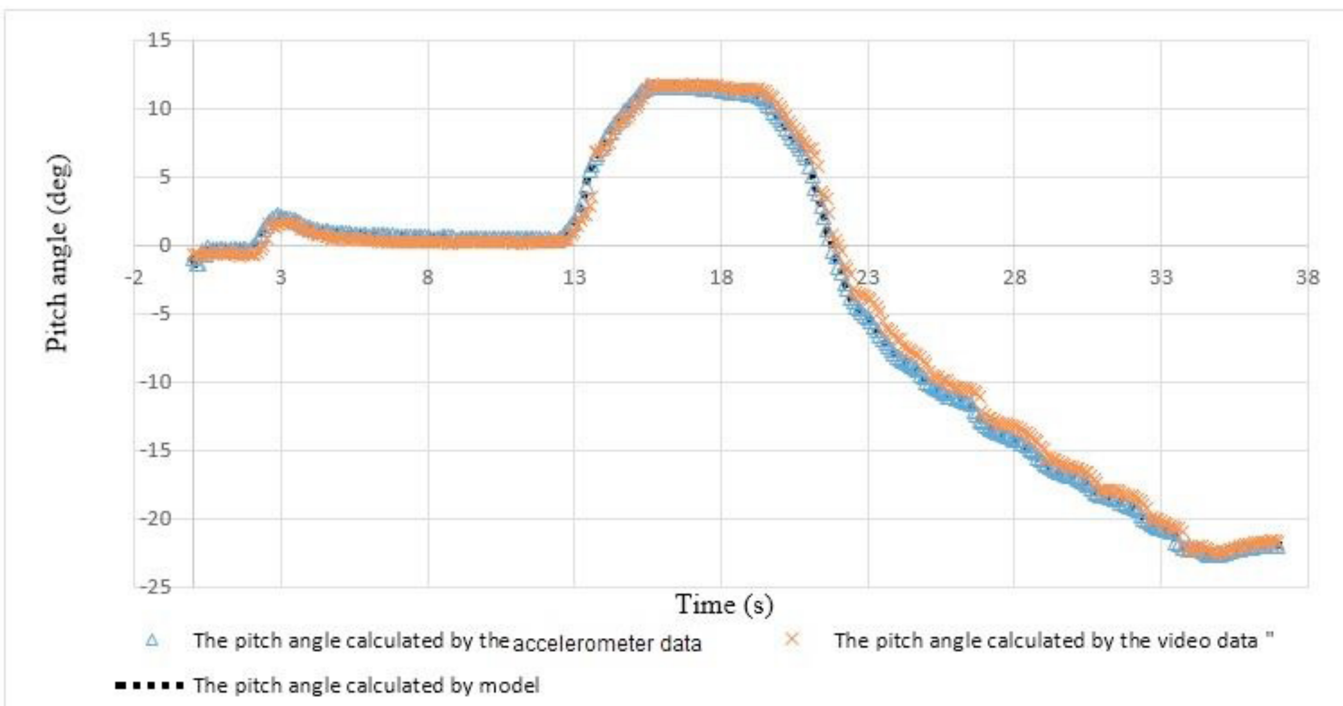

Fig. 12. Results of semi-natural experiment.

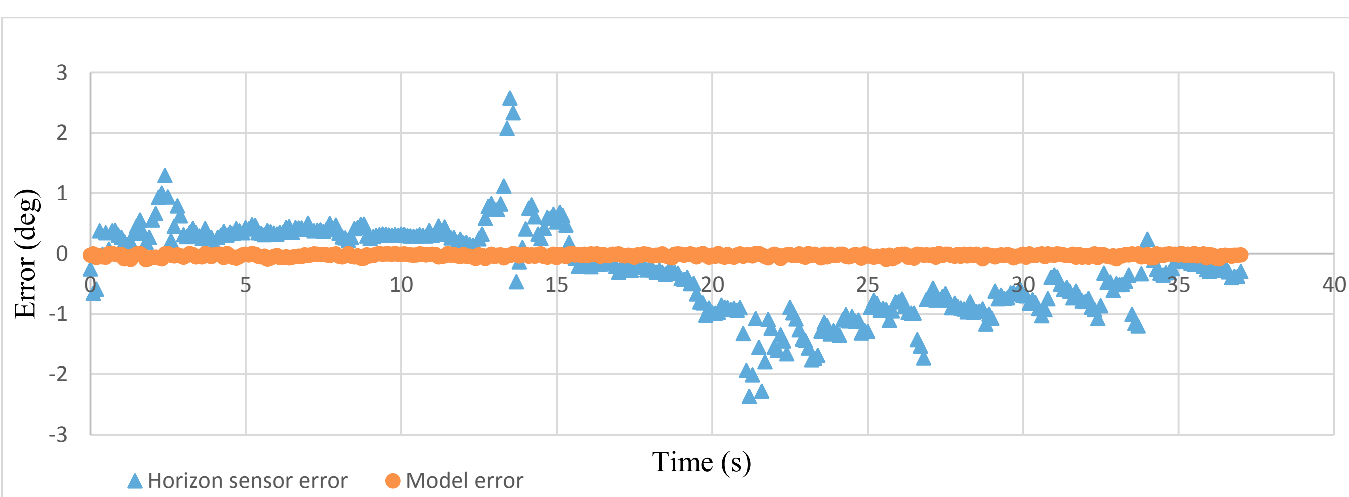

Fig. 13. Experimental error.