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# MODES OF MOTION OF SOYUZ ORBITAL STAGE AFTER PAYLOAD SEPARATION AT CARRYING OUT OF SHORT-TERM RESEARCH EXPERIMENTS

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The uncontrolled motion of Soyuz orbital stage after spacecraft separation is discussed. Two versions of motion are considered. The first variant is the stage motion after utilization of the jet nozzle; the second variant is the stage motion without utilization of a jet nozzle. The stochastic model of initial conditions of angular motion is formulated. The influence of the gravitational and the aerodynamic moments on the motion around its mass center is considered. Movement features of the orbital stage shown in preservation of certain angular orientation during time, sufficient for the successful decision of navigation-communication problems at carrying out of short-term research experiments on Soyuz carrier rocket orbital stage after separation of mail payload are revealed.

## INTRODUCTION

In the case of the orbital stage utilization for carrying out of experiments it is necessary to provide data transfer to Earth. For this purpose, first of all, the motion of the orbital stage which perform uncontrolled motion after payload separation, have to study. The components of the angular velocity vector of the orbital stage in the body-fixed frame of reference at the moment of separation of the payload are found[1] in the result of motion simulation:

$$\omega_x = -(2.5 \pm 0.3)\ \text{deg/s},$$

$$\omega_y = (0.0 \pm 2.5)\ \text{deg/s}, \tag{1}$$

$$\omega_z = (0.0 \pm 2.5)\ \text{deg/s}.$$

The jet nozzle of the oxidizer tank is turned on over 0.7 sec after separation of the payload from orbital stage. This creates an additional force that leads the orbital stage from the spacecraft. The components of the angular velocity vector of the orbital stage in the body-fixed frame of reference after activity of the jet nozzle are obtained following[1]:

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$$\omega_x = (93 \pm 120) \text{ deg/s},$$

$$\omega_y = (-11 \pm 117) \text{ deg/s}, \quad (2)$$

$$\omega_z = (2 \pm 69) \text{ deg/s}.$$

The deviations of components of the angular velocity vector from the average are adduced in the supposition about a normal law of distribution of values of the probability density. Deviations are calculated as the tripled value of a standard deviation.

**REGULAR PRECESSION**

In the beginning motion of the orbital stage around its mass center, neglecting action of outside forces, is considered. It is supposed, that kinetic energy of gyration of the orbital stage is much more than activity of the outside forces stipulated by influence of Sun pressure, atmosphere, gravitation and magnetic terrestrial fields. The orbital stage is considered as dynamically symmetrical rigid body (inertia moments concerning transverse axes are equal: $I_y = I_z = I_n$ ). The rotary motion of the orbital stage is represented like regular precession at which the longitudinal axis passing through a mass center, describes a circular cone concerning the kinetic moment vector (angular momentum) $\vec{K}_0$. This one saves the constant direction in the space (Figure 1), where $\alpha_k$ is the angle between the axis of symmetry and the vector $\vec{K}_0$ (the half-angle cone of precession), $\dot{\psi}$ is the angular velocity of precession, $\dot{\varphi}$ is the angular velocity of proper rotation. The initial direction of transversal component angular velocity may be supposed as the random quantity distributed uniformly in the interval from 0 up to 2π.

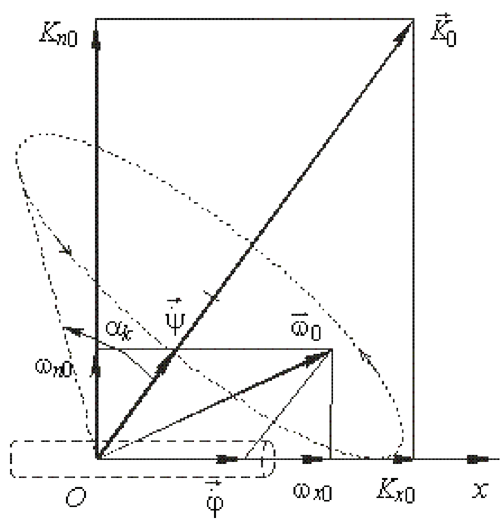



**Figure 1. Regular precession.**

The values of the half-angle cone of precession, the angular velocity of precession, the angular velocity of proper rotation are calculated by formulas[2]

$$\alpha_k = \arcsin\left(\frac{K_{n0}}{K_0}\right),$$

$$\dot{\psi} = \frac{I_x \omega_{x0}}{I_n \cos\alpha_k}, \qquad (3)$$

$$\dot{\varphi} = \frac{(I_n - I_x)\omega_{x0}}{I_n},$$

where $K_0 = \sqrt{K_{x0}^2 + K_{n0}^2}$ is the module of the kinetic moment; $K_{x0} = I_x \omega_{x0}$, $K_{n0} = I_n \omega_{n0}$ - are longitudinal and transversal components of kinetic moment; $\omega_{x0}$, $\omega_{n0} = \sqrt{\omega_{y0}^2 + \omega_{z0}^2}$ are longitudinal and transversal components of angular velocity.

Let's compare parameters of the regular precession implemented in case of the utilization of the jet nozzle with parameters of the regular precession implemented in case without of the utilization of the jet nozzle.

Statistical simulation (10000 numerical experiments) has been carried out under formulas (3) for definition of statistical performances of distribution of the random variables of the half-angle cone of precession $\alpha_k$, the angular velocity of precession $\dot{\psi}$, the angular velocity of proper rotation $\dot{\varphi}$ and the transversal angular velocity $\omega_n$. In this case, the distribution laws of components of angular velocity were received according to (1) and (2). Longitudinal and transversal inertia moments of the orbital stage were considered as random variables uniformly distributed within the corresponding range.

The probability density function for the half-angle cone of precession in the case of utilization of the jet nozzle is shown in the Figure 2, and in case without of utilization of the jet nozzle is shown in the Figure 3. Statistical performances of allocations for angles and angle velocities ($\bar{x}$ is the arithmetic mean, $\sigma_x$ is the standard deviation) are shown in Table 1.

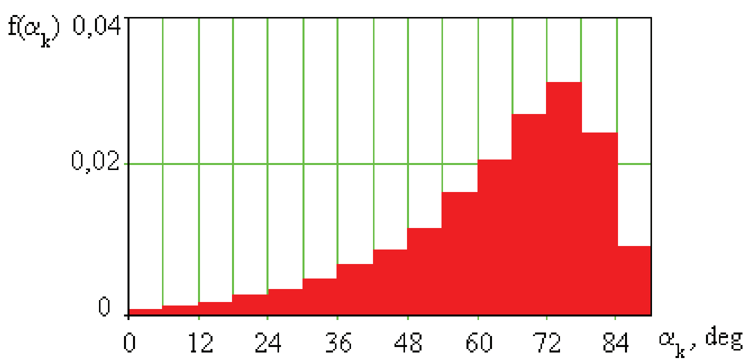



**Figure 2. Probability density function of the half-angle cone of precession in the case of utilization of the jet nozzle.**

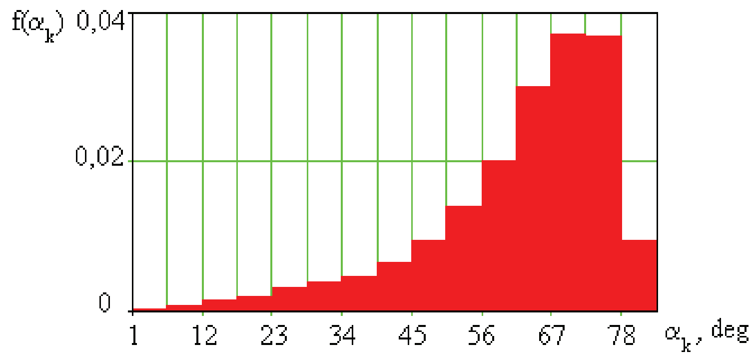



**Figure 3. Probability density function of the half-angle cone of precession in the case without of utilization of the jet nozzle.**

**Table 1. Statistical performances of allocations.**

| Value / Mode | Half-angle cone of precession | | Angular velocity of precession | | Angular velocity of proper rotation | | Transversal angular velocity | |
|---|---|---|---|---|---|---|---|---|
| | $\bar{\alpha}_k$, deg | $\sigma_{\alpha_k}$, deg | $\bar{\dot{\psi}}$, deg/s | $\sigma_{\dot{\psi}}$, deg/s | $\bar{\dot{\varphi}}$, deg/s | $\sigma_{\dot{\varphi}}$, deg/s | $\bar{\omega}_n$, deg/s | $\sigma_{\omega_n}$, deg/s |
| Utilization of the jet nozzle | 64.0 | 16.6 | 44.4 | 22.3 | 77.3 | 33.1 | 40.7 | 22.6 |
| Without utilization of the jet nozzle | 62.7 | 14.4 | 1.15 | 0.49 | 2.08 | 0.10 | 1.04 | 0.55 |

Apparently in both cases statistical performances of allocations of the half-angle cone of precession are close, but statistical performances of allocations of the angular velocity of precession, the angular velocity of proper rotation and the transversal angular velocity are differed more than on the order.

If the external moments are very small, the kinetic moment vector saves own value and a direction in the space. Kinetic energy saves own value too. However, if there are mobile elements on the orbital stage then kinetic energy of their relative motions is transferred in heat and dissipated. In this case the half-angle cone of precession is changed. The rotation about the axis of the least inertia moment of orbital stage is unstable. Thus, the longitudinal angular velocity will decrease, and the transversal angular velocity to increase. Increase of the half-angle cone of precession as the result of motion of the transversal axis of orbital stage to the kinetic moment vector. The motion of orbital stage will move to a stable state. The stable state is rotation concerning the axes of the greatest inertia moment ( $\alpha_k \rightarrow 90\,\text{deg}$ ). It is assumed that, the move of mobile elements not lead essential change of inertia moments. In this case it is possible to define approximately the limiting value of transversal angular velocity $\omega_{nk}$ and value of dispersed energy $\Delta T$ under formulas[3]:

$$\omega_{nk} = \sqrt{\left(\frac{I_x}{I_n}\right)^2 \omega_{x0}^2 + \omega_{n0}^2},$$

$$\Delta T = -\frac{1}{2}\left(1 - \frac{I_x}{I_n}\right) I_x \omega_{x0}^2.$$

The values of the statistical performances of allocation (the arithmetic mean and the standard deviation) for the limiting value of transversal angular velocity $\omega_{nk}$ are found in the case of utilization of the jet nozzle:

$$\bar{\omega}_{nk} = 45.0\,\text{deg/s}, \quad \sigma_{\omega_{nk}} = 21.0\,\text{deg/s};$$

and in the case without of utilization of the jet nozzle:

$$\bar{\omega}_{nk} = 1.15\,\text{deg/s}, \quad \sigma_{\omega_{nk}} = 0.49\,\text{deg/s}.$$

**FORCED PRECESSION**

Let's consider motion of the orbital stage around its center of mass under action of external forces on the low near-circular orbit with the maximal altitude of 245 km and the minimal altitude of 193 km, which use for the "Progress" cargo space vehicle. Let us consider the influence of the gravitation and aerodynamic moments, the influence of magnetic moment not taken into account for investigation.

Let us determine the position of the kinetic moment vector relatively of the mass center velocity vector by the angle $\alpha_v$, and the position of the longitudinal axis of orbital stage relatively of the mass center velocity vector by the spatial angle of attack $\alpha_s$ (see Figure 4).

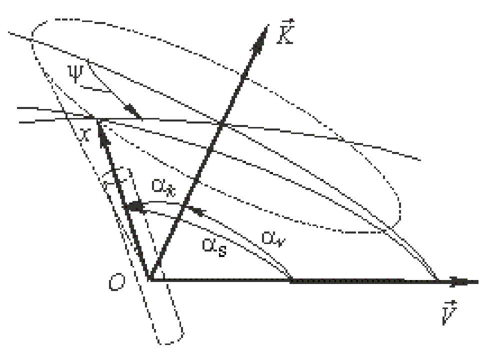



**Figure 4. Orientation of the kinetic moment vector and the longitudinal axis of orbital stage concerning velocity vector.**

Let's compare value of the gravitation moment with value of the aerodynamic moment, which is capsizing in a considered case. The influence of moment of dissipative aerodynamic forces is neglected. For inertial, centering, aerodynamic parameters are taken their mean values. Change of ratio of maximum value of the aerodynamic moment to maximum value of the gravitation moment from the altitude for two limiting values of atmospheric density[4] (night atmosphere at the minimal solar activity and day atmosphere at the maximum solar activity) are shown in Table 2.

**Table 2. Change of ratio of maximum value of the aerodynamic moment to maximum value of the gravitation moment from the altitude.**

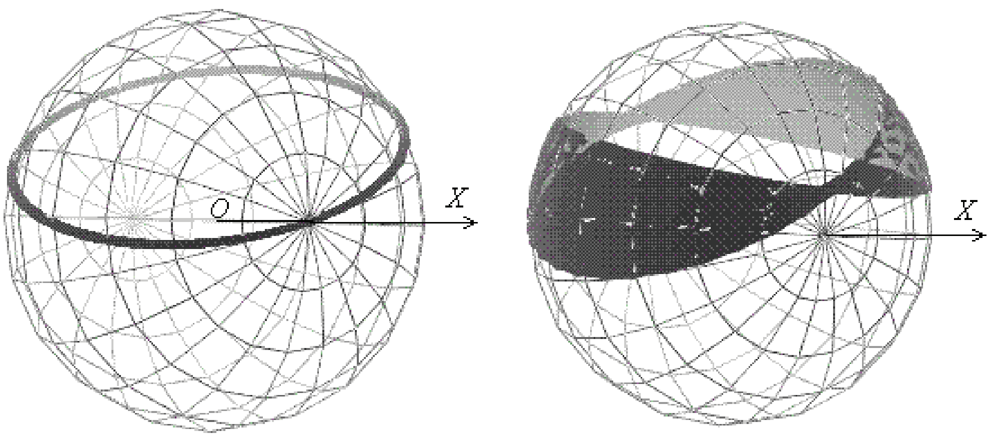
| Altitude, km | 250 | 225 | 200 | 175 | 150 | 125 | 100 |
|---|---|---|---|---|---|---|---|
| Night atmosphere at the minimal solar activity | 0.4 | 1.1 | 3.1 | 11.5 | 49.3 | 321 | 8021 |
| Day atmosphere at the maximum solar activity | 2.3 | 3.9 | 7.2 | 16.1 | 46.5 | 319 | 8021 |

So the aerodynamic moment is the moment which defines the orbital stage dynamics. His influence on motion of orbital stage around its mass center is exhibited in following. The aerodynamic moment aspire to combine the longitudinal axis of orbital stage with the incoming airflow direction. However it's pitching motion is counteracted by gyroscopic forces leading to the forced precession of the kinetic moment vector concerning the mass center velocity vector. The kinetic moment vector deviates to the side of the vector of the aerodynamic moment.

Let's compare the character of angular motion implemented in case of utilization of the jet nozzle with the character of angular motion implemented in case without of utilization of the jet nozzle. Calculations have been carried out by a numerical integration of the complete set of the differential equations describing motion of a mass center and motion concerning of a mass center. For geometrical, aerodynamic, inertial, centering parameters were set their mean values. The atmospheric density as day atmosphere for the maximum solar activity was determined.

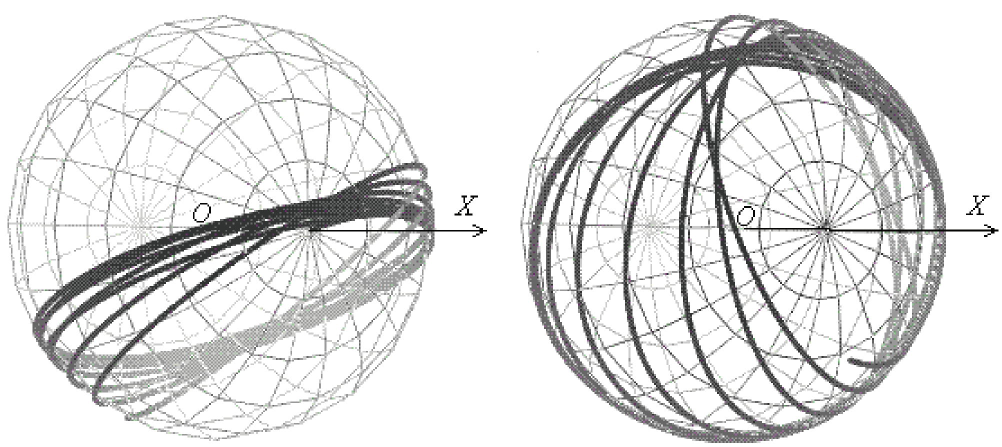
In the case of utilization of the jet nozzle the attitude motion of orbital stage close to the regular precession down to the moment of atmospheric entry (flight altitude $H$=125 km) owing to large value of kinetic moment. The trajectory of the end of the longitudinal axis of orbital stage on the unit sphere concerning the inertial reference frame with the beginning in center of mass (axis $OX$ is directed on us) is shown in Figure 5. In the initial time the inertial reference frame coincides with orbital reference frame, and axis $OX$ coincides with the transversal one. As is seen in Figure 5a, for the flight altitude $H$=200 km the orbital stage execute the regular precession (the trajectory of the end of the longitudinal axis is shown on the time interval $\Delta t$=2650 s). For the flight altitude $H$=125 km (see Figure 5b) the forced precession is become noticeable ($\Delta t$=1325 s).

In the case of don't utilization of the jet nozzle the value of kinetic moment is much less and the attitude motion is close to the regular precession up to flight altitudes $H$=180 km. Then, in accordance with descent the capsizing aerodynamic moment intensively will increase and the longitudinal axis of orbital stage execute the motion concerning of incoming airflow direction. The trajectory of the end of the longitudinal axis of orbital stage on the unit sphere concerning the inertial reference frame with the beginning in center of mass (axis $OX$ is directed on us) is shown in Figure 6. As seen in Figure 6a, for the flight altitude $H$=200 km the attitude motion of the orbital stage close to the regular precession (interval of time $\Delta t$=2650 s). For the flight altitude $H$=180 km (see Figure 6b) the forced precession is become noticeable ($\Delta t$=2650 s). The trajectory of the end of the longitudinal axis of orbital stage on the unit sphere concerning the trajectory reference frame with the beginning in center of mass (the incoming airflow is directed on us) for the flight altitude $H$=125 km ($\Delta t$=2650 s) is shown in Figure 7.




(a) (b)

**Figure 5. Trajectory of the end of the longitudinal axis of the orbital stage on the unit sphere concerning the inertial reference frame in case of utilization of the jet nozzle: (a) flight altitude $H$ = 200 km, time interval $\Delta t$ = 2650 s; (b) flight altitude $H$ = 125 km, time interval $\Delta t$ = 1325 s.**




(a) (b)

**Figure 6. Trajectory of the end of the longitudinal axis of the orbital stage on the unit sphere concerning the inertial reference frame in case of don't utilization of the jet nozzle: (a) flight altitude $H$ = 200 km, time interval $\Delta t$ = 2650 s; (b) flight altitude $H$ = 180 km, time interval $\Delta t$ = 2650 s.**

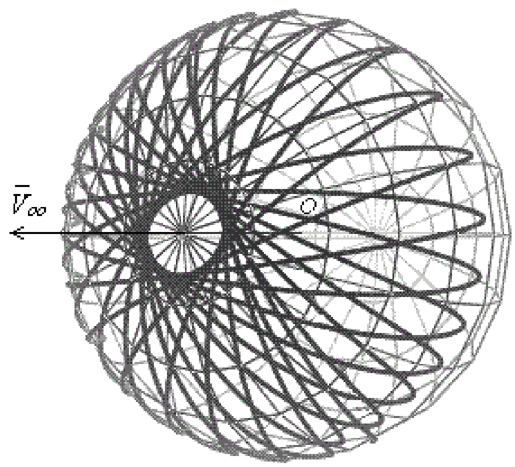



**Figure 7. Trajectory of the end of the longitudinal axis of the orbital stage on the unit sphere concerning the trajectory reference frame in case of don't utilization of the jet nozzle (flight altitude *H* = 125 km, time interval *Δt* = 2650 s).**

For estimation of the possibility of data transmission via the Globalstar satellites system was carried out statistical simulation (10000 numerical experiments). It was assumed that the angle of the cone of radio visibility from orbital stage is $\delta = 180\,\mathrm{deg}$ and the orbital stage performs a regular precession (the half-angle cone of precession is $\alpha_k = 64\,\mathrm{deg}$, the angular velocity of precession is $\dot{\psi} = 44.4\,\mathrm{deg/s}$). In the result the cone of angle of constant view from orbital stage decreases up to $\delta_c = 52\,\mathrm{deg}$. The initial position of the communication satellite in the orbital plane is considered as random variables uniformly distributed in the interval [0, π/3], the angle between the communication satellites in the orbit plane was taken π/3. It was assumed that the orbital stage moves in the plane of the satellites and the kinetic moment vector (constant direction in space), is also located in the plane of motion of the satellites. Figure 8 shows the probability density function of communication sessions (in minutes) with the Globalstar satellites on the orbital turn. This result can take into account a lower limit estimate of the time of communication sessions with the Globalstar satellites on the orbital turn.

The obtained results allow to determine the probability of the possibility of communication session holding on the turn of orbital stage. For example, the probability of a communication session length of at least 3 minutes (enough to transmit the data set of around 1.5 Mbit, using the GSP-1620 satellite modem) is 0.933.

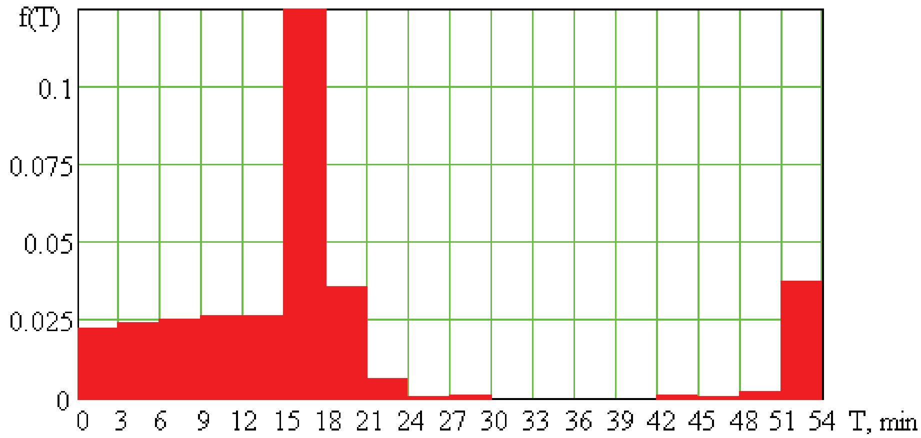



**Figure 8. Probability density function of communication sessions with the Globalstar satellites system on the orbital turn.**

## CONCLUSIONS

Features of the dynamics of movement of orbital stage of carrier rocket Soyuz after separation of payload revealed. In both cases (utilization and don't utilization of jet nozzle) statistical performances of the half-angle cone of precession are close, but statistical performances of the allocations of the angular velocity of precession, the angular velocity of proper rotation and the transversal angular velocity are differed more than on the order. The motion of the orbital stage allows to carry out the short-term experiments, as it is possible for sufficient data transmission.

## ACKNOWLEDGEMENT

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