ALGORITHM FOR REORIENTATION OF THE CUBESAT NANOSATELLITES *

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Abstract

Keywords: nanosatellite, reorientation, inverse dynamic problem, program trajectory, algorithm

Now nanosatellites of the CubeSat format are most popular among the other nanosatellites. The nanosatellites of the CubeSat format can be used for various purposes, for example, for carrying out the experiments on geophysical field studying, for remote sensing of the Earth with the low/average resolution, etc. The attitude control system plays an important role in the solution of goal-oriented problems in orbit. It allows reorienting the nanosatellite from any initial attitude in the required one over a specified period of time. In the report the algorithm for reorientation of the CubeSat nanosatellite based on the principle of the solution of the inverse dynamic problems taking into account action of the aerodynamic and gravitational torques is presented, as the program trajectory the polynomial of the fifth degree is chosen. The absolute amount values of the control torque for the various duration of the turn process are compared, the effect of the attitude determination error on the accuracy of the reorientation process, and also neglect of the aerodynamic torque in the reorientation algorithm that affected the reorientation process accuracy are considered at orbit heights that are typical for nanosatellite standard missions (300-450 km).

Introduction

Around the world among the spacecraft functioning now small and very small spacecraft attract considerable interest. It is connected with the possibility to reduce the spacecraft cost when preserving efficiency of the mission performance due to reduction of mass-inertial characteristics of spacecraft hardware configuration and with the prospects of application of orbital groups of such spacecrafts launched to the orbit by one carrier. Taking into account the modern level of space technologies, the special attention from scientific and technical community is paid to spacecrafts weighing from 1 to 10 kg – nanosatellites. They can be used for various purposes, for example, for research of the impact of radiation on the hardware components, for testing new devices in space conditions, for remote sensing of the Earth with the low/average resolution, for geophysical field monitoring, etc.

For the majority of the listed problems it is required to solve the nanosatellite reorientation problem. The reorientation problem is solved by attitude control system, which allows reorienting the nanosatellite from unspecified initial attitude in the required final attitude for the given period. Complication of experiments with the use of nanosatellites requires the reorientation algorithm development for providing of the required angular position.

In research the algorithm for reorientation of the CubeSat [1] nanosatellite based on the principle of the inverse dynamic problem solution [2] is provided, there are reasons for a choice of the program trajectory for making the turn of the nanosatellite, results of mathematical simulation of the reorientation process are given and analyzed.

Mathematical statement of problem of the nanosatellite reorientation

The mutual position of the orbital and body frames of reference can be described by quaternion [3]:

$$\mathbf{\Lambda}(t) = \left(\lambda_0(t),\, \lambda_1(t),\, \lambda_2(t),\, \lambda_3(t)\right) \in \mathbf{R}^4,$$

the components of which should fulfill condition of normalization:

$$\left|\mathbf{\Lambda}(t)\right|^2 = \lambda_0^2(t) + \lambda_1^2(t) + \lambda_2^2(t) + \lambda_3^2(t) = 1\,. \quad (1)$$

For the description of motion about center of mass the dynamic and kinematic equations are used [3]:

$$\frac{d\mathbf{K}}{dt} + \boldsymbol{\omega} \times \mathbf{K}_0 = \mathbf{M}_0 + \mathbf{U}\,, \quad (2)$$

$$2\dot{\mathbf{\Lambda}} = \mathbf{\Lambda} \circ \boldsymbol{\omega} - \mathbf{\Lambda} \circ \boldsymbol{\omega}_{орб}\,, \quad (3)$$

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where $\boldsymbol{\omega}$ is the absolute angular rate, $\mathbf{K}_0 = \mathbf{I}\boldsymbol{\omega}$ is the kinetic torque vector, $\mathbf{I}$ is the inertia tensor, $\mathbf{M}_0$ is the main torque of outside forces, $\mathbf{U}$ is the control torque vector; $\boldsymbol{\omega}_{орб}$ is the orbital angular rate.

At orbital flight in the set orbit, a nanosatellite is mainly influenced by the gravitational and aerodynamic disturbing torques [4]. It is necessary to consider that for orbit heights typical for nanosatellite standard missions (300-450 km) influence of the aerodynamic torque is more than gravitational one [5].

Nanosatellite reorientation algorithm

Values of functions $\lambda_i(t)$ and their first and second derivatives determine the initial and final system states and value of control torque in them for any program trajectory at the ends of interval $[0, t_к]$ [6]

$$\dot{\boldsymbol{\Lambda}}(t)\big|_{t=0} = \dot{\boldsymbol{\Lambda}}_0 = 0.5\boldsymbol{\Lambda}_0 \circ \left(\boldsymbol{\omega}_0 - \boldsymbol{\omega}_{орб0}\right),$$

$$\ddot{\boldsymbol{\Lambda}}(t)\big|_{t=0} = \ddot{\boldsymbol{\Lambda}}_0 = 0.5\left(\dot{\boldsymbol{\Lambda}}_0 \circ \boldsymbol{\omega}_0 + \boldsymbol{\Lambda}_0 \circ \dot{\boldsymbol{\omega}}_0 - \dot{\boldsymbol{\Lambda}}_0 \circ \boldsymbol{\omega}_{орб0} - \boldsymbol{\Lambda}_0 \circ \dot{\boldsymbol{\omega}}_{орб0}\right).$$

If $t = t_к$, similarly find that

$$\dot{\boldsymbol{\Lambda}}(t)\big|_{t=t_k} = \dot{\boldsymbol{\Lambda}}_k = -0.5\boldsymbol{\Lambda}_k \circ \boldsymbol{\omega}_{орбk},$$

$$\ddot{\boldsymbol{\Lambda}}(t)\big|_{t=t_k} = \ddot{\boldsymbol{\Lambda}}_k = -0.5\left(\dot{\boldsymbol{\Lambda}}_k \circ \boldsymbol{\omega}_{орбk} + \boldsymbol{\Lambda}_k \circ \dot{\boldsymbol{\omega}}_{орбk}\right).$$

As there are 6 boundary conditions, for realization of the program trajectory the polynomial not less than the fifth degree should be used [6]:

$$\mu_i(t) = \frac{-12(\lambda_i - \lambda_i^*) - 6\dot{\lambda}_i t_k - 6\dot{\lambda}_i^* t_k - \ddot{\lambda}_i t_k^2 + \ddot{\lambda}_i^* t_k^2}{2t_k^5} t^5 +$$

$$+\frac{30(\lambda_i - \lambda_i^*) + 16\dot{\lambda}_i t_k + 14\dot{\lambda}_i^* t_k + 3\ddot{\lambda}_i t_k^2 - 2\ddot{\lambda}_i^* t_k^2}{2t_k^4} t^4 +$$

$$+\frac{-20(\lambda_i - \lambda_i^*) - 12\dot{\lambda}_i t_k - 8\dot{\lambda}_i^* t_k - 3\ddot{\lambda}_i t_k^2 + \ddot{\lambda}_{i0}^* t_k^2}{2t_k^3} t^3 + 0.5\ddot{\lambda}_i t^2 + \dot{\lambda}_i t + \lambda_i.$$

The program trajectory fulfills condition of normalization (1) and looks like [6]:

$$\lambda_i(t) = \frac{\mu_i(t)}{\sqrt{\sum_{i=0}^{3} \mu_i^2(t)}}, \quad \dot{\lambda}_i(t) = \frac{d\lambda_i(t)}{dt}, \quad \ddot{\lambda}_i(t) = \frac{d^2\lambda_i(t)}{dt^2},$$

where $i = \overline{0, 3}$.

The analytical equation of the control torque is derived with the use of equations (2) and (3):

$$\mathbf{U} = \mathbf{I} \cdot \left(2 \cdot \boldsymbol{\Lambda}^{-1}(t) \circ \ddot{\boldsymbol{\Lambda}}(t) + \dot{\boldsymbol{\omega}}_{орб}(t) - 2 \cdot \boldsymbol{\Lambda}^{-1}(t) \circ \dot{\boldsymbol{\Lambda}}(t) \circ \boldsymbol{\Lambda}^{-1}(t) \circ \dot{\boldsymbol{\Lambda}}(t)\right) +$$

$$+\left(2 \cdot \boldsymbol{\Lambda}^{-1}(t) \circ \dot{\boldsymbol{\Lambda}}(t) + \boldsymbol{\omega}_{орб}(t)\right) \times \mathbf{I} \cdot \left(2 \cdot \boldsymbol{\Lambda}^{-1}(t) \circ \dot{\boldsymbol{\Lambda}}(t) + \boldsymbol{\omega}_{орб}(t)\right) - \mathbf{M}.$$

Mathematical simulation results

The case of the spatial turn from any initial position $\alpha_n = 180°$, $\gamma_a = 60°$, $\phi_n = -30°$ in the required final one $\alpha_n = 0°$, $\gamma_a = 0°$, $\phi_n = 0°$ is considered. The analysis of influence of turn time on the absolute amount of control torque value for this case of the turn has been carried out. Duration of time of the turn is 50, 500 and 5000 seconds (fig. 1, *a-c*).

As it is seen from fig. 1, absolute amounts of control torque value for time of the turn of 50, 500 and 5000 seconds do not exceed $|\mathbf{M}|_{50} \le 9.8 \cdot 10^{-3}\ N \cdot m$, $|\mathbf{M}|_{500} \le 8 \cdot 10^{-5}\ N \cdot m$, $|\mathbf{M}|_{5000} \le 1.8 \cdot 10^{-6}\ N \cdot m$ respectively.

The error of attitude control also depends on the error of determination of initial conditions of motion about the center of mass. In fig. 2 dependence of the error of control (spatial angle of attack) on the errors of initial data is shown.

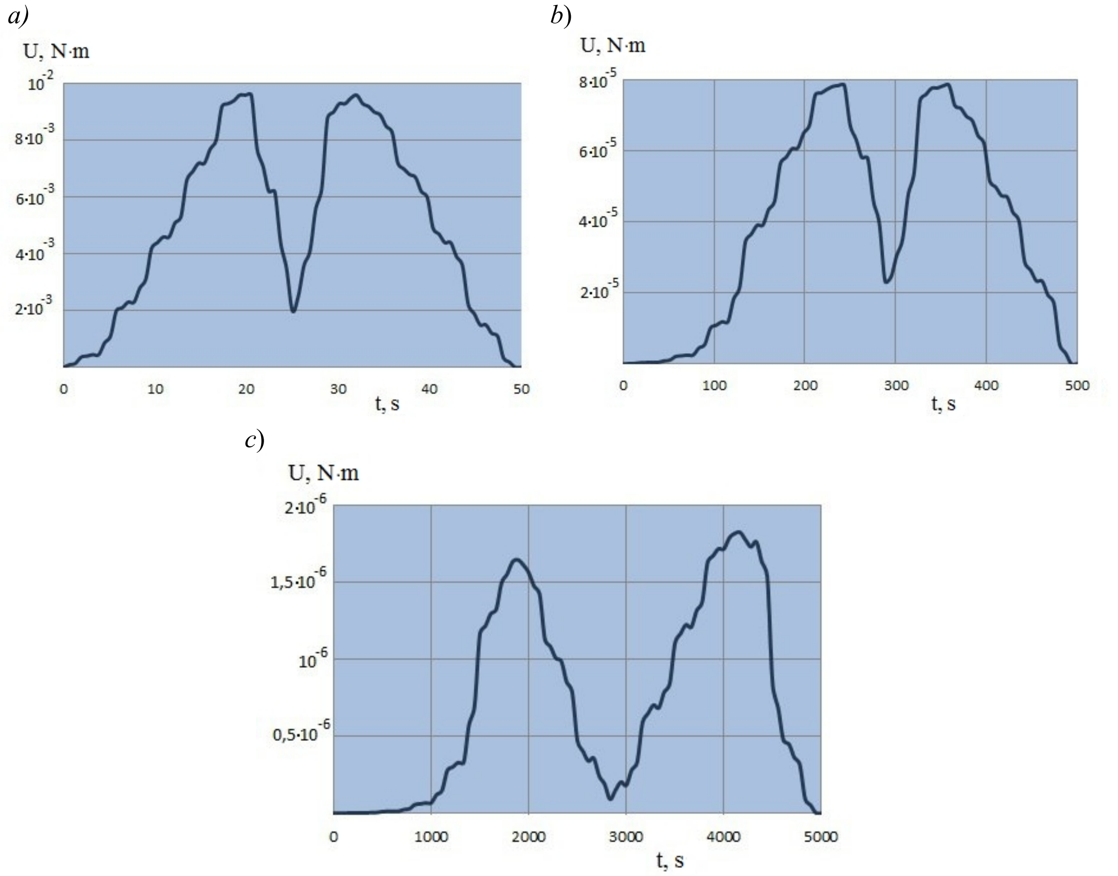

Fig. 1. The absolute amounts of control torque value at the turn: *a*) in 50 seconds; *b*) in 500 seconds; *c*) in 5000 seconds

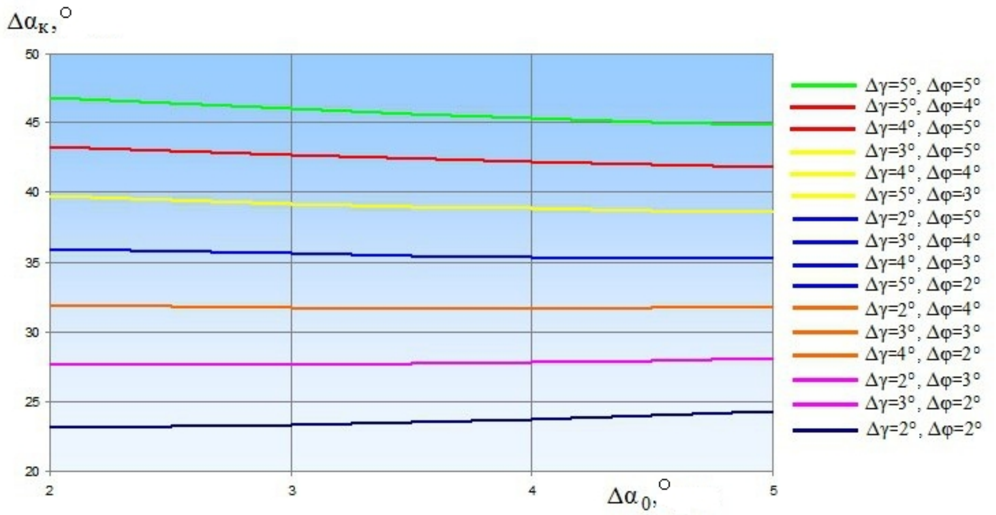

Fig. 2. Dependence of the angle of attack error at spatial turn on the error of attitude determination system

As it is seen from fig. 2, the given algorithm of the nanosatellite reorientation imposes strict requirements on the accuracy of the nanosatellite attitude determination.

While calculating the control on a long time, it is necessary to take into account the external torques acting on the nanosatellite.

Let's consider the control constructed without the aerodynamic torque. During numerical simulation of the motion about the center of mass the aerodynamic torque is used. The spatial case of the turn from any initial attitude $\alpha_n = 180°$, $\gamma_a = 60°$, $\phi_n = -30°$, in the required final attitude $\alpha_n = 0°$, $\gamma_a = 0°$, $\phi_n = 0°$ has been considered taking into account the control torque, but without the aerodynamic torque (fig. 3).

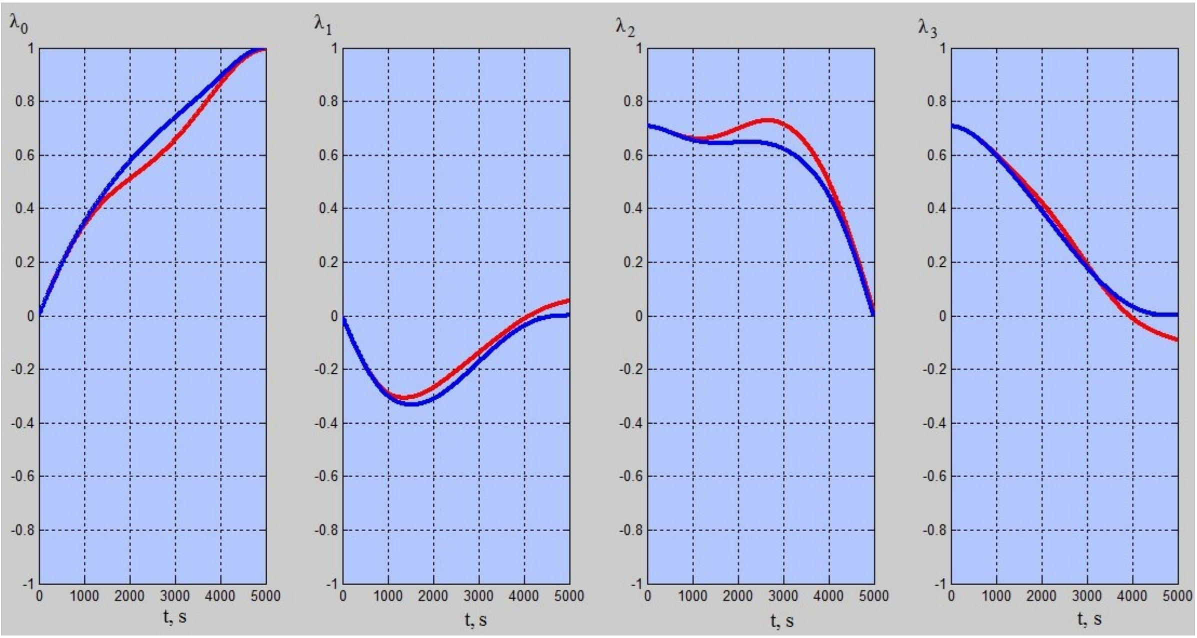

Fig. 3. The turn from one horizontal position in other horizontal position
without the aerodynamic torque during realization of control

As can be seen from fig. 3 the aerodynamic torque at heights that are typical for nanosatellite standard missions affects significantly on the motion about the center of mass.

Conclusions

As a result of the research, the reorientation algorithm taking into account the action of the aerodynamic and gravitational torques on the basis of the principle of the inverse dynamic problem solution has been developed. The algorithm analysis by means of mathematical simulation has been conducted. Comparison of the absolute amount values of the control torque for various duration of the turn process has been carried out, the effect of the attitude determination error on the accuracy of the reorientation process and also neglect of the aerodynamic torque in the reorientation algorithm that affected the reorientation process accuracy have been considered.

In prospect the presented algorithm will be used for reorientation of nanosatellites of the SamSat family developed at Samara University.

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