

## The Adaptive Algorithm of Separation Program for a Nanosatellites Cluster From Space Platform Executed Uncontrolled Motion

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### Abstract

Nowadays for nanosatellites it is used a piggyback launch with a space platform, for example, an upper (orbital) stages of rockets. Process of separating nanosatellites is one of the most critical stages of their operation. Emergency processes of the separation or a wrong choice of separation program's parameters leads to the failure of mission objectives. It can lead to dangerous approaches of nanosatellites to each other or to a space platform. The aim of this work is to develop an adaptive algorithm for choosing of separation parameters for a nanosatellites group. Algorithm should allow us to recalculate the separation program quickly in case of an emergency (for example, failure of an attitude control system) on a space platform. The algorithm is based on the use of probabilistic analytical models of the relative motion and uses a priori information about the possible range of angular velocities that can obtain a space platform in case of failure of the attitude control system. The algorithm allows to determine the areas of values of time delay and separation velocity for each nanosatellite using the requirements of safety relatively to the space platform and the minimum time delay between the separation of nanosatellites to provide a safe relative motion and distance no more than required. To select separation parameters nanosatellites from the found areas it is proposed to use a minimax criterion of a distance between them. The algorithm consists two stages. At the first stage after determining a set of nanosatellites separation parameters that have satisfied all the necessary conditions it should be determine the maximum distance between the nanosatellites. At the second stage it should be selected those separation parameters for which the maximum value of the distance between the nanosatellites are minimal. The developed algorithm of selection separation program for a group of nanosatellites separated from an undirected space platform is based on the use of analytical models and provides guaranteed separation parameters for safety in case of emergency. The algorithm can be implemented on board of a spacecraft platform.

**Keywords:** nanosatellite cluster, orbital stage, nanosatellite separation, formation flight

### Nomenclature

$V_x, V_y, V_z$  – separation velocity projections;  
 $E_{V_x}, E_{V_y}, E_{V_z}$  – expected values of separation velocity projections;  
 $Var_{V_x}, Var_{V_y}, Var_{V_z}$  – variance of separation velocity projections;  
 $x, y, z$  – coordinates of a nanosatellite relative to a space platform;  
 $E_x, E_y, E_z$  – expected values of nanosatellite's coordinates;  
 $Var_x, Var_y, Var_z$  – variance of nanosatellite's coordinates;  
 $V_i, \Delta t_i$  – separation velocity and time delay of  $i$ -th nanosatellite;  
 $r_i$  – the distance between each nanosatellite and a space platform;  
 $r_{ij}$  – the distance between two nanosatellites;  
 $i=1, \dots, n-1, j=1, \dots, n, i>j, n$  – nanosatellite's number;  
 $\sigma$  – deviation of components of the transverse angular velocity, which are considered independent and distributed according to the normal law;  
 $r^*$  – the radius of safe area;

$r^{**}$  – the maximum distance between two nanosatellites;  
 $t$  – current time;  
 $P^* = 0,997$  – the required value of the probability,  
 $K_{V_x V_y}$  – covariance of separation velocity projections;  
 $\omega = \sqrt{\frac{\mu}{R^3}}$  – angular orbital velocity;  
 $\mu = 398602 \text{ km}^3/\text{s}^2$  – the Earth gravitational parameter;  
 $R$  – radius-vector of a space platform;  
 $P_x = \rho V_s^2 \cdot \Delta Q$  – the projection of the aerodynamic acceleration on the transversal direction;  
 $\Delta Q$  – the difference between ballistic coefficients of space platform and a nanosatellite;  
 $\rho$  – the atmospheric density;  
 $V_s$  – the free-stream velocity;  
 $T$  – the certain time interval.

### Acronyms/Abbreviations

NS – Nanosatellite;  
OS – Orbital stage.

## 1. Introduction

Nowadays, there has been a trend in the world of increasing the number of nanosatellites (NS) launches in comparison to the total number of spacecraft being launched in orbits. At the same time, the range of tasks, for the implementation of which it is necessary to launch nanosatellites clustered together, is expanding, (Figure 1).



Fig. 1. Nanosatellite cluster orbiting

However, orbiting of a nanosatellite cluster is a significant problem, which usually solves by a piggyback launch (additional payload) with a main payload from a space platform. For a number of reasons, some platforms can have uncontrolled motion relatively to its center of mass with random parameters (for example, orbital stages (OS) of carrier rockets after separation of the main payload or space platforms in case of failures in their control systems). Direction of additional payload separation cannot be predicted because of the uncontrolled motion of the space platform. There are many papers devoted to the separation of nanosatellites [1-3]. Those papers do not consider problems related to the choice of the parameters for separation program for a group of NS from an uncontrolled space platform (which may be OS). Also they do not take into account the random nature of its motion.

In this paper, it is proposed an adaptive algorithm for selecting parameters for the separation of a nanosatellites group. This algorithm makes it possible to recalculate quickly the separation program in the case of its uncontrolled motion. For example, this algorithm is applicable if there is an emergency situation on the space platform, when there is no time to change the

separation program from the Earth and there is no possibility to use large computing power on board.

## 2. Mathematical model of relative motion used for the adaptive algorithm

Here and after, as an example, the orbital (upper) stage of a carrier rocket is considering as the space platform from which the nanosatellites are separated. The orientation of the OS was determined in a probabilistic formulation using the regular precession model presented in paper [4]. On the basis of this model, the authors of paper [5] obtained analytical models for the distribution function of the NS separation velocity projections in the orbital coordinate system (the center of the system is at the center of mass of the OS, the  $y$  axis is directed along the radius vector from the Earth center to OS, the  $x$  axis is along the direction of motion, the  $z$  axis supplements the system to the right). The mathematical expectations and variances of the projections of the NS separation speed are obtained:

$$E_{V_x} = V + V \exp\left(\frac{-\pi^2}{2\sigma^2 \Delta t^2}\right) \cdot Q(V_x),$$

$$E_{V_y} = 0,$$

$$E_{V_z} = 0.$$

$$Q(V_x) = \int_{-V}^V q(V_x) dV_x,$$

$$q(V_x) = \exp\left(\frac{-\arccos^2(V_x/V)}{2\sigma^2 \Delta t^2}\right) dV_x.$$

$$\text{Var}_{V_x} = V^2 + V^2 \exp\left(\frac{-\pi^2}{2\sigma^2 \Delta t^2}\right) \cdot E_{V_x}^2 - B_x(V_x),$$

$$\text{Var}_{V_y} = \frac{1}{2\sigma \Delta t V \sqrt{2\pi}} B_y(V_y),$$

$$\text{Var}_{V_z} = \frac{1}{2\sigma \Delta t V \sqrt{2\pi}} B_z(V_z).$$

$$B_x(V_x) = 2 \int_{-V}^V V_x \exp\left(\frac{-\arccos^2(V_x/V)}{2\sigma^2 \Delta t^2}\right) dV_x,$$

$$B_y(V_x) = \int_{-V}^V V_y^2 \exp\left(\frac{-V_y^2}{2\sigma^2 \Delta t^2}\right) dV_y,$$

$$B_z(V_x) = \int_{-V}^V V_z^2 \exp\left(\frac{-V_z^2}{2\sigma^2 \Delta t^2}\right) dV_z.$$

These analytical expressions are obtained as a quadrature. Using the operation of the mathematical expectation and the variance calculating of the coordinates of the relative motion (for the linearized model of the relative motion [6]) and assuming that the random variables  $V_x$  and  $V_y$  are independent it is obtained:

$$E_x = E_{V_x} \left( \frac{4}{\omega} \sin(\omega t) - 3t \right) + \frac{4P_x}{\omega^2} (1 - \cos(\omega t)) - \frac{3}{2} P_x t^2,$$

$$E_y = E_{V_x} \left( \frac{2}{\omega} - \frac{2}{\omega} \cos(\omega t) \right) + \frac{2P_x}{\omega} \left( t - \frac{\sin(\omega t)}{\omega} \right), \quad (1)$$

$$E_z = 0.$$

$$Var_x = Var_{V_x} \cdot \left( \frac{4}{\omega} \sin(\omega t) - 3t \right)^2 + Var_{V_y} \cdot \left( \frac{2}{\omega} \cos(\omega t) - \frac{2}{\omega} \right)^2$$

$$Var_y = Var_{V_x} \cdot \left( \frac{2}{\omega} - \frac{2}{\omega} \cos(\omega t) \right)^2 + Var_{V_y} \cdot \left( \frac{1}{\omega} \sin(\omega t) \right)^2 \quad (2)$$

$$Var_z = Var_{V_z} \cdot \frac{\sin^2(\omega t)}{\omega}$$

The obtained analytical expressions for mathematical expectations and variances of the coordinates of the position of the NS relative to the OS in orbital coordinate system will be used in the algorithm for approximate estimation of the nanosatellite separation parameters.

It is noticed that, as an example, it is considered the separation of NS from OS. In this case, the orbit of the NS will not exceed 250 km. In this case, the used mathematical model of relative motion takes into account the influence of the atmosphere, but does not take into account that the gravitational field is not centered.

### 3. The adaptive algorithm for nanosatellite separation parameters estimation

The adaptive algorithm for estimation of the NS separation parameters is developed on the basis of probabilistic analytical models of relative motion (1) and (2). The separation parameters that determine the NS separation are velocity and time delay of the separation. The algorithm based on determination of the time delay  $\Delta t_i$  and the separation velocity  $V_i$  of the  $i$ -th NS. These parameters determined from the condition of providing a safe motion in relation to the OS (3). Also it is necessary to determine the value of the minimum time delay  $\Delta t_{n-m}$  between the separation of nanosatellites with equal separation velocities from the condition of providing a safe motion in relation to each other and from the condition to provide a given distance between nanosatellites (4):

$$r_i(t) \geq r^*, \quad (3)$$

$$r^{**} \leq r_{ij}(t) \leq r^*. \quad (4)$$

To implement the algorithm for estimating the NS separation parameters it is necessary to choose the separation time delay and velocity of each NS for calculating of mathematical expectations and variances of the coordinates of the position of the NS relative to the OS. Next it is necessary to satisfy conditions (3) and (4).

After determining the set of separation parameters that satisfy conditions (3) and (4), it is suggested to use the minimax criterion for the distance between NS:

$$\min_{\Delta t_i} \max_{i,j,t} r_{i-j} \left| P(r_{i-j}(t) \leq r^{**}) \geq P^*, t \in T \quad (5) \right.$$

$$i > j, i = 1..n, j = 1..n.$$

The application of the criterion: after determining the set of nanosatellites separation sequences (for those which are included in a group) that satisfy all necessary conditions, the maximum distances between NS are determined. As a result, it is selected a separation sequence which provide the maximum value of the distance between any pair of NS to be minimum.

The scheme of the algorithm for estimating the NS separation parameters is shown in Figure 2.

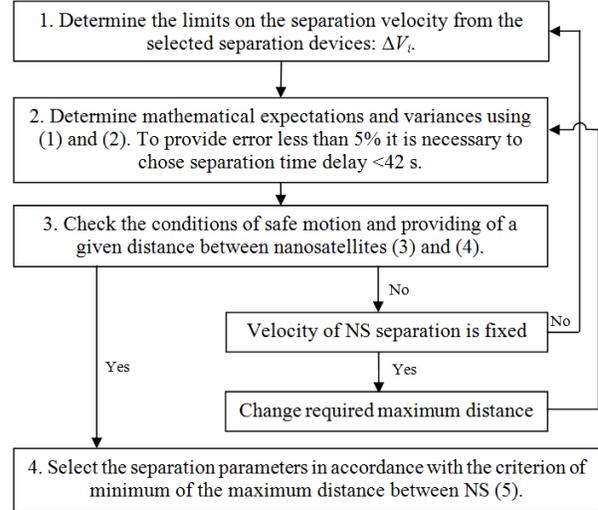


Fig. 2. The scheme of the algorithm for estimating the NS separation parameters

Thus, the developed algorithm is based on:

1) analytical models of relative motion with the assumption that the random variables of the NS separation projections  $V_x$  and  $V_y$  are independent. It means that the correlation moment  $K_{V_x V_y}$  was assumed to be 0;

2) the mathematical model of relative motion along a circular orbit in the central field of gravity with influence of the atmosphere.

For example, the areas of the NS separation parameters obtained from analytical models of relative motion are shown in figure 3.

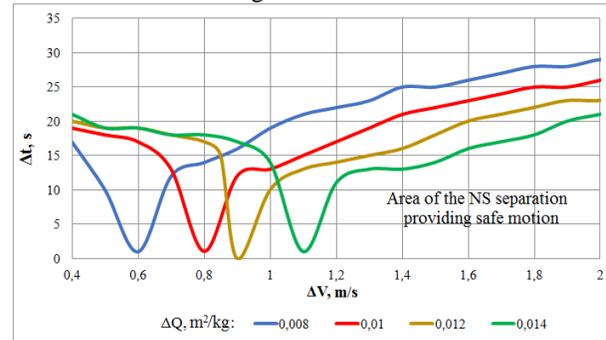


Fig. 3. Areas of the NS separation providing safe motion in relation to the OS

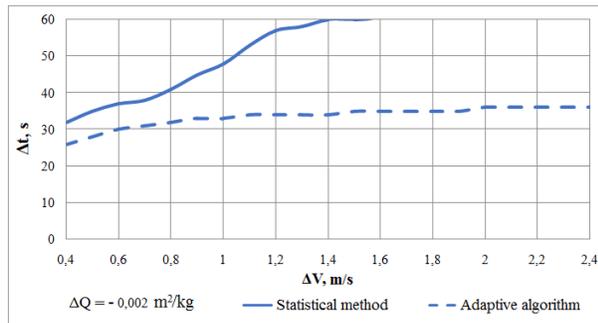
The areas below the lines in Figure 3 show the NS separation parameters, which provide their safe motion in relation to the OS in dependence on the difference in the ballistic coefficients of the NS and OS ( $\Delta Q$ ) at 220 km altitude. In this case, the probability of safe motion is 0.997.

#### 4. Estimation of the accuracy of the adaptive algorithm

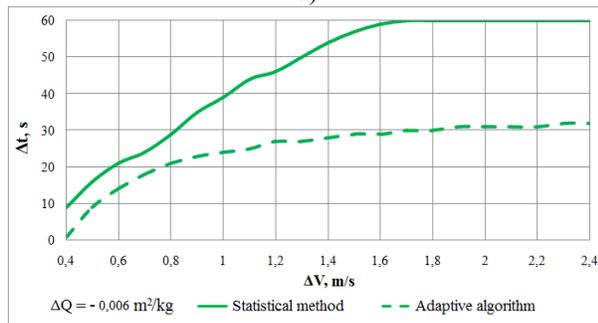
A comparison is made between the application of the developed adaptive algorithm for selecting the NS separation parameters based on analytical models of relative motion with a statistical method.

The statistical method was carried out using more accurate models of relative motion. The applied mathematical model is a system of exact differential equations that take into account the elliptical orbit and the influence of the atmosphere. More details of the methodology and results of the statistical study are described in paper [7].

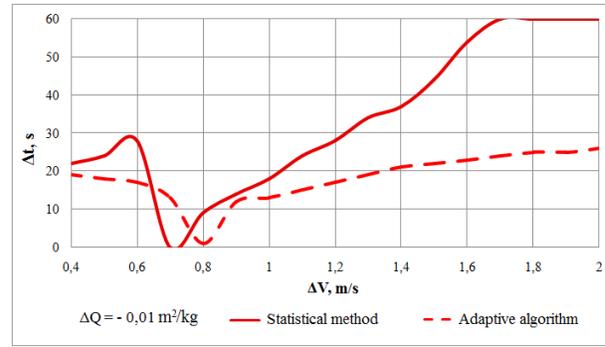
For this comparison it was obtained areas of NS separation parameters to provide safe motion in relation to the OS of the Soyuz carrier rocket. Those areas obtained with the developed algorithm and using the statistical method (figure 4).



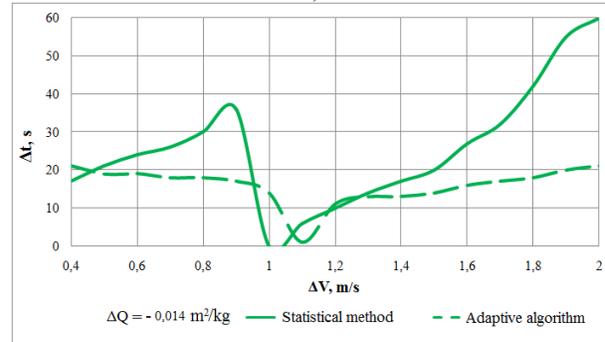
a)



b)



B)



Γ)

Fig. 4. Comparison of results obtained by developed algorithm and using the statistical method

It can be seen from the figure 4 that the application of the analytical adaptive algorithm makes more stringent restrictions on the choice of the NS separation parameters.

#### 5. Example of adaptive algorithm application

As an example, it is consider the application of the developed algorithm in the case of four NS separation from the orbital stage of the launch vehicle "Soyuz". In this case, the first three of them form a cluster performing a common task, and for them it is required to provide a distance no more than 10 km in the time interval of two turns, and the fourth NS has its own mission. The nanosatellites from the cluster are separated using a typical separation device, hence the velocity of their separation is fixed.

As known from paper [8], the orbital stage after separation of the main payload performs a non-oriented motion around the mass center with random initial parameters. In this regard, it is assumed that the required probability of fulfilling the claimed requirements is 0.997.

The values of the difference in the ballistic coefficients are assumed:  $\Delta Q_{ns2-1} = 0,001 \text{ m}^2/\text{kg}$ ;  $\Delta Q_{ns3-1} = 0,002 \text{ m}^2/\text{kg}$ ;  $\Delta Q_{ns3-2} = 0,001 \text{ m}^2/\text{kg}$ ;  $\Delta Q_{ns4-1} = -0,002 \text{ m}^2/\text{kg}$ ;  $\Delta Q_{ns4-2} = -0,003 \text{ m}^2/\text{kg}$ ;  $\Delta Q_{ns4-3} = -0,004 \text{ m}^2/\text{kg}$ ;  $\Delta Q_{os-1} = -0,006 \text{ m}^2/\text{kg}$ ;  $\Delta Q_{os-2} = -0,007 \text{ m}^2/\text{kg}$ ,  $\Delta Q_{os-3} = -0,008 \text{ m}^2/\text{kg}$ ,

$\Delta Q_{os-4} = -0,004 \text{ m}^2/\text{kg}$ , where the indices mean the numbers of each NS.

Selected separation device provides the same separation velocity for each of the three NS forming the cluster:  $\Delta V_1 = \Delta V_2 = \Delta V_3 = 2 \text{ m/s}$ . For the 4th NS, which is deployed separately, it was select the separation velocity of typical device  $\Delta V_4 = 1 \text{ m/s}$ , which provides the safe motion.

Needed conditions have been defined using equations (1) and (2). The separation conditions, providing the safe motion of each NS in relation to the OS:

$$\begin{cases} 5c \leq \Delta t_1 < 40c \\ 5c \leq \Delta t_2 < 40c \\ 5c \leq \Delta t_3 < 40c \\ 5c \leq \Delta t_4 < 25c \end{cases}$$

The separation conditions, providing the safe motion of each NS in relation to each other:

$$\begin{cases} \Delta t_2 - \Delta t_1 \geq 6c \\ \Delta t_3 - \Delta t_1 \geq 5c \\ \Delta t_3 - \Delta t_2 \geq 6c \end{cases}$$

The separation conditions, providing distance between NS no more than 10 km:

$$\begin{cases} \Delta t_2 - \Delta t_1 \leq 12c \\ \Delta t_3 - \Delta t_1 \leq 11c \\ \Delta t_3 - \Delta t_2 \leq 12c \end{cases}$$

Applying the minimax criterion of the distance between the NS (in this case the minimum value of the maximum distance is provided with the minimum delay time of separation between each NS), we obtain:  $\Delta t_1 = 5 \text{ s}$ ,  $\Delta t_3 = 10 \text{ s}$ ,  $\Delta t_2 = 16 \text{ s}$ .

For the fourth NS, which is not part of the cluster, the time delay must satisfy only the separation condition, which provides the safe motion of each NS in relation to the OS, so it can be deployed immediately after the cluster is separated, for example, with a delay  $\Delta t_4 = 17 \text{ s}$ .

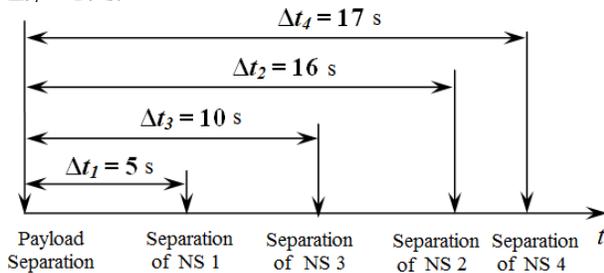


Fig. 5. Example of a separation program

The selected separation program provides safe NS motion in relation to each other and to the OS with a probability of 0.997. Also it provides the distance between three NS (that must implement a formation flight) at a distance no more than 10 km for two turns of flight.

## 6. Conclusions

Thus, the developed algorithm is suitable for determining the approximate parameters of the NS separation, when it is required to make a first estimation of the necessary separation parameters or in the case when it is impossible to carry out a statistical simulation of the relative motion. For example, after launching a carrier rocket, various emergency situations are possible, which may require to change the parameters of the separation program in real time. Such situations, for example, include jamming one of the NS in deployer, motion in a wrong orbit or wrong parameters of the space platform motion relative to its center of mass.

In this case, due to limited time for decision making and limited computational resources, it is required to apply an algorithm that does not require large computing powers and with which it would be possible to correct the generated program of nanosatellite separation.

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