ESTIMATION OF THE NANOSATELLITE ATTITUDE AND ANGULAR RATE BY ANALYZING THE NAVIGATION SPACECRAFT GEOMETRICAL VISIBILITY USING THE CONTROLLABLE PATTERN OF NAVIGATION ANTENNA

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According to the statistics of launches of spacecrafts of nanoclass (nanosatellites), CubeSat is the most popular format of nanosatellites. Nanosatellites of this format are widely used for the solution of different problems of scientific and application-oriented assignment. For the correct interpretation of results of the experiments made on nanosatellites it is necessary to know the full vector of phase variables of the nanosatellite: coordinates, rates, angles and angular rates, i.e. it is necessary to solve the problem of navigation and attitude determination. It is advisable to solve the problem of navigation with the use of the user navigation equipment, and it is possible to determine the attitude approximately by the analysis of geometrical visibility of navigation spacecrafts of GLONASS and GPS systems. Changing width of the antenna pattern of the user navigation equipment it is possible to increase the accuracy of attitude determination of an axis (or spatial attitude, in case of three antennas), and solving the problem in dynamics, it is possible to estimate angular rate of the nanosatellite. In research the algorithm of estimation of attitude of the nanosatellite axis, along which antennas of the user navigation equipment with the changing pattern width are located, is provided. Requirements to the control law of the pattern of phase center of the antenna of the user navigation equipment are formulated.

***Keywords:** nanosatellite, attitude, geometrical visibility, navigation antenna, algorithm*

I. INTRODUCTION

Use of the usernavigation equipment (UNE), working on signals of the global navigation satellite systems (GNSS) - GLONASS (Russia), GPS (the USA), Galileo (the European Union), allows solving the problem of navigation (determination of parameters of the center of mass motion) independently and with the fine precision. Therefore, the UNE is the obligatory element of spacecraft onboard systems now.

The existing UNE use two types of measurements: phase and code. The user navigation equipment using phase measurements can be also applied to determine the spacecraft attitude based on the interferometry principles [1-3].

The UNE based on the phase measurements has both advantages – high accuracy of the navigation and attitude problem solution, and disadvantages – the high cost, the necessity of the phase ambiguity resolution, the constructive complexity of realization on the spacecraft, as the accuracy of the attitude determination at phase measurements considerably depends on the size of antenna base.

Considering the small sizes of nanosatellites, the UNE realization with the multiantenna system with the use of the interferometry principles is difficult. At the same time, it should be noted that not all satellite radionavigational information in the UNE is used in full.In papers [4, 5] it is shown that using information on the geometrical visibility of the navigation spacecrafts (NS) of the GNSS and knowing their location in space it is possible to estimate the spatial attitude of the nanosatellite axis, on which the UNE antenna is installed. If three antennas are installed on the mutually perpendicular axes, it is possible to estimate the nanosatellite spatial attitude on one-shot measurements. The attitude estimation error at the same time will not exceed 15 deg [4, 5].

Changing the width of the UNE antenna directivity diagram it is possible to increase the accuracy of the attitude determination of an axis (or spatial attitude, in case of three antennas), and solving the problem in dynamics, it is possible to estimate the nanosatellite angular rate.

II. MATHEMATICAL STATEMENT OF PROBLEM OF THE NANOSATELLITE MONOAXIAL ATTITUDE DETERMINATION

At nanosatellite attitude determination problem statement the following hypotheses are accepted:

- on the nanosatellite two UNE antennas on the axis OX_1 of the body frame of reference in the positive direction ($\mathbf{A}_1^+$) and in the negative one ($\mathbf{A}_1^-$) are installed;
- width of the UNE antenna pattern can change with any discretization, which for definiteness we will consider equal to 1 °.

At statement and solution of the nanosatellite attitude problem it is convenient to enter the right orthogonal frames of reference (trihedron) with the center in the object center of mass: the body frame of reference $OX_1Y_1Z_1$ (BFR) and the orbital frame of reference $OX_2Y_2Z_2$ (OFR) [6].

The problem of spatial attitude determination of the nanosatellite axis according to the analysis of spatial location of visible/invisible NS of GLONANSS/GPS GNSS is formulated based on the UNE information:

- $x, y, z, \dot{x}, \dot{y}, \dot{z}$ are parameters of the nanosatellite center of mass motion;
- x_i, y_i, z_i are coordinates of NS of GLONANSS/GPS GNSS ($i = \overline{1, N}$);
- $\mathbf{A}_1^{\pm} = \left(x_1^{\pm}, y_1^{\pm}, z_1^{\pm}\right)$ is vector of the direction cosines of the UNE antenna phase centers in the BFR;

it is required to find estimation of the nanosatellite axis spatial attitude, i.e. estimation of the direction cosines of the UNE antenna phase centers located on the nanosatellite longitudinal axis taking into account a condition of normalization of the direction cosines of the UNE antenna phase centers

$$\left(x_2^{\pm}\right)^2 + \left(y_2^{\pm}\right)^2 + \left(z_2^{\pm}\right)^2 = 1. \qquad (1)$$

III. ALGORITHM OF THE NANOSATELLITE MONOAXIAL ATTITUDE DETERMINATION ACCORDING TO THE ANALYSIS OF THE NS GEOMETRICAL VISIBILITY

The algorithm of the spacecraft attitude determination is based on the use of information on the spatial attitude of NS of GLONASS and GPS GNSS [2] and includes the following steps:

- Formation of array of coordinates of visible/invisible NS of GLONASS and GPS GNSS for the time solving the problem of the nanosatellite longitudinal axis attitude determination;
- Recalculation of ranges to visible/invisible NS from the absolute frame of reference to the OFR by means of matrix:

$$\mathbf{A} = \begin{bmatrix} -\sin u\cos\omega - \cos u\sin\omega\cos i & \cos u\cos\omega - \sin u\sin\omega\cos i & -\sin\omega\sin i \\ -\sin u\sin\omega + \cos u\cos\omega\cos i & \cos u\sin\omega + \sin u\cos\omega\cos i & \cos\omega\sin i \\ \cos u\cos i & \sin u\sin i & -\cos i \end{bmatrix}.$$

- Rejection of NS shaded by the Earth. The Earth shading condition:

$$z_{2k} < 0 \text{ and } \left|z_{2k}\right| > \cos\left(\arcsin\frac{R_з}{R_з + h}\right), k = \left.\begin{matrix}\overline{1, N_{ГЛ}} \\ 1, N_{GPS}\end{matrix}\right\}.$$

- Search of estimation of the vector of the antenna phase center direction cosines from condition of minimum of the objective function (*F*) and attitude angle determination.

The objective function showing the geometrical sense of NS visibility will take a form:

$$\begin{aligned} F\left(x_2^{\pm}, y_2^{\pm}, z_2^{\pm}\right) = \\ = \alpha_1 \cdot \Bigg[\sum_{i=1}^{N^- - N_{ИСК}^-} \left(x_2^- \cdot x_{2i} + y_2^- \cdot y_{2i} + z_2^- \cdot z_{2i} - 1\right)^2 + \\ + \sum_{j=1}^{N^+ - N_{ИСК}^+} \left(x_2^+ \cdot x_{2j} + y_2^+ \cdot y_{2j} + z_2^+ \cdot z_{2j} - 1\right)^2 \Bigg] + \qquad (2) \\ + \alpha_2 \cdot \Bigg[\sum_{k=1}^{N_{ИСК}^-} \left(x_2^- \cdot x_{2k} + y_2^- \cdot y_{2k} + z_2^- \cdot z_{2k} - c^-\right)^2 + \\ + \sum_{s=1}^{N_{ИСК}^+} \left(x_2^+ \cdot x_{2s} + y_2^+ \cdot y_{2s} + z_2^+ \cdot z_{2s} - c^+\right)^2 \Bigg], \end{aligned}$$

where $c^{\pm} = \cos(\gamma^{\pm})$ is cosine of the UNE antenna half-angle $\mathbf{A}_1^{\pm}$; α_i are weight coefficients fulfilled condition $\alpha_1 + \alpha_2 = 1$; N^-, N^+ are numbers of visible NS by $\mathbf{A}_1^-$ and $\mathbf{A}_1^+$ antennas respectively; $N_{ИСК}^-, N_{ИСК}^+$ are numbers of NS that are “invisible” due to changing the width of the UNE antenna pattern.

Difference of the objective function (2) from described in papers [3,4] consists in existence of regularized summand, which appears as a result of NS transition from category "visible" to category "invisible" because of changing the width of the UNE antenna pattern.

As it was noted above, for use of the controllable mask it is necessary to install two UNE antennas on one axis in opposite directions. Vectors of the direction cosines of the UNE antenna phase centers located on one axis are connected by equation:

$$A_2^+\left(x_2, y_2, z_2\right) = -A_2^-\left(x_2, y_2, z_2\right), \qquad (3)$$

where $A_2^+(x_2, y_2, z_2)$ is vector of the UNE antenna phase center (located in the positive direction of the OX axes) in the OFR;

$A_2^-(x_2, y_2, z_2)$ is vector of the UNE antenna phase center (located in the negative direction of the OX axes) in the OFR.

Considering (3) and provided that width of the UNE antenna pattern changes with equal discretization, the objective function (2) will take a form

$$F\left(x_2, y_2, z_2\right) =$$

$$= \alpha_1 \cdot \left[\sum_{i=1}^{N^- - N^-_{\text{ИСК}}} \left(x_2 \cdot x_{2i} + y_2 \cdot y_{2i} + z_2 \cdot z_{2i} + 1\right)^2 + \right.$$

$$\left. + \sum_{j=1}^{N^+ - N^+_{\text{ИСК}}} \left(x_2 \cdot x_{2j} + y_2 \cdot y_{2j} + z_2 \cdot z_{2j} - 1\right)^2 \right] +$$

$$+ \alpha_2 \cdot \left[\sum_{k=1}^{N^-_{\text{ИСК}}} \left(x_2 \cdot x_{2k} + y_2 \cdot y_{2k} + z_2 \cdot z_{2k} + c\right)^2 + \right.$$

$$\left. + \sum_{s=1}^{N^+_{\text{ИСК}}} \left(x_2 \cdot x_{2s} + y_2 \cdot y_{2s} + z_2 \cdot z_{2s} - c\right)^2 \right],$$

Vector of the UNE antenna phase center in the OFR, i.e. vector $A_2^+(x_2, y_2, z_2)$ is found from minimum of the objective function (4) with the following normalization of the found vector. Minimization of the objective function (4) comes to the solution of the linear equation system

$$\mathbf{M} \cdot \mathbf{A}_2 = \mathbf{b},$$

where $\mathbf{M}$ is the symmetric matrix of the size 3×3 with elements:

$$m_{11} = \sum_{i=1}^{N^-} x_{2i}^2 + \sum_{j=1}^{N^+} x_{2j}^2 + \sum_{k=1}^{N^-_{\text{ИСК}}} x_{2k}^2 + \sum_{s=1}^{N^+_{\text{ИСК}}} x_{2s}^2,$$

$$m_{22} = \sum_{i=1}^{N^-} y_{2i}^2 + \sum_{j=1}^{N^+} y_{2j}^2 + \sum_{k=1}^{N^-_{\text{ИСК}}} y_{2k}^2 + \sum_{s=1}^{N^+_{\text{ИСК}}} y_{2s}^2,$$

$$m_{33} = \sum_{i=1}^{N^-} z_{2i}^2 + \sum_{j=1}^{N^+} z_{2j}^2 + \sum_{k=1}^{N^-_{\text{ИСК}}} z_{2k}^2 + \sum_{s=1}^{N^+_{\text{ИСК}}} z_{2s}^2,$$

$$m_{12} = \sum_{i=1}^{N^-} x_{2i} \cdot y_{2i} + \sum_{j=1}^{N^+} x_{2j} \cdot y_{2j} + \sum_{k=1}^{N^-_{\text{ИСК}}} x_{2k} \cdot y_{2k} + \sum_{s=1}^{N^+_{\text{ИСК}}} x_{2s} \cdot y_{2s},$$

$$m_{13} = \sum_{i=1}^{N^-} x_{2i} \cdot z_{2i} + \sum_{j=1}^{N^+} x_{2j} \cdot z_{2j} + \sum_{k=1}^{N^-_{\text{ИСК}}} x_{2k} \cdot z_{2k} + \sum_{s=1}^{N^+_{\text{ИСК}}} x_{2s} \cdot z_{2s},$$

$$m_{23} = \sum_{i=1}^{N^-} y_{2i} \cdot z_{2i} + \sum_{j=1}^{N^+} y_{2j} \cdot z_{2j} + \sum_{k=1}^{N^-_{\text{ИСК}}} y_{2k} \cdot z_{2k} + \sum_{s=1}^{N^+_{\text{ИСК}}} y_{2s} \cdot z_{2s},$$

elements of the $\mathbf{b}$ vector are:

$$b_1 = \sum_{j=1}^{N^+} x_{2j} - \sum_{i=1}^{N^-} x_{2i} + \sum_{s=1}^{N^+_{\text{ИСК}}} x_{2s} \cdot c - \sum_{k=1}^{N^-_{\text{ИСК}}} x_{2k} \cdot c, b_2 =$$

$$= \sum_{j=1}^{N^+} y_{2j} - \sum_{i=1}^{N^-} y_{2i} + \sum_{s=1}^{N^+_{\text{ИСК}}} y_{2s} \cdot c - \sum_{k=1}^{N^-_{\text{ИСК}}} y_{2k} \cdot c,$$

$$b_3 = \sum_{j=1}^{N^+} z_{2j} - \sum_{i=1}^{N^-} z_{2i} + \sum_{s=1}^{N^+_{\text{ИСК}}} z_{2s} \cdot c - \sum_{k=1}^{N^-_{\text{ИСК}}} z_{2k} \cdot c.$$

IV. MATHEMATICAL SIMULATION RESULTS

For estimation of efficiency of application of the UNE antenna controllable pattern the extensive simulation was carried out.

During simulation the following hypotheses have been accepted:

- orbital structures of GLONASS and GPS GNSS are taken on 01.03.2017;
- nanosatellite orbit is circular with height of 400 km.

We will understand the spatial angle between true vector of the UNE antenna and the found estimation as error of the nanosatellite axis attitude determination.

In Fig. 1 dependence of the error of the nanosatellite axis attitude determination on width of the UNE antenna pattern is shown.

In Fig. 2 dependence of the error of the nanosatellite axis attitude determination on weight coefficient α_2 is shown.

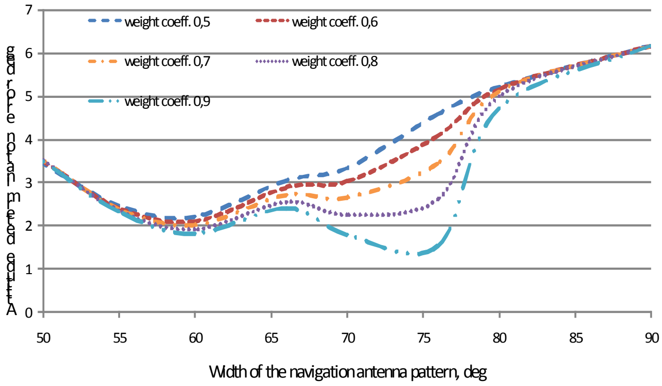

Fig. 1. Dependence of the error of the nanosatellite axis attitude determination on width of the UNE antenna pattern

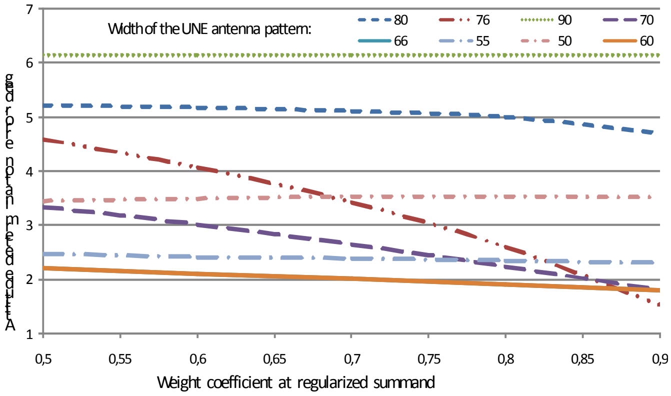

Fig. 2. Dependence of the error of the nanosatellite axis attitude determination on weight coefficient α_2.

V. CONCLUSIONS

As is clear from fig. 1 and fig. 2, introduction to the objective function of regularized summand generated by change of width of the UNE antenna pattern gives to increase in accuracy of the nanosatellite monoaxial attitude determination by 3-4 times. Introduction of regularized summand does not influence on the algorithm computing complexity. For obtaining the best result, it is necessary to control reduction of width of the UNE antenna pattern.

For search of the nanosatellite spatial attitude it is necessary to install 2 or 3 pairs of the UNE antennas on the corresponding axes and to apply the stated algorithm to each pair of the UNE antennas.

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