

# DEVELOPMENT AND INVESTIGATION OF ALGORITHMS FOR DETERMINING RELATIVE NAVIGATION AND ORIENTATION BASED ON DISTANCE MEASUREMENTS

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**Abstract** — In research the problems of nanosatellite relative navigation and attitude determination at formation flight are considered. For relative navigation and attitude determination the algorithms based on methods of range radionavigation have been developed. The algorithm field of applicability is formulated. Estimates of the algorithm method errors are obtained and recommendations about their practical use are formulated.

**Keywords**—nanosatellite; formation flight; relative navigation; relative attitude, algorithm

## I. INTRODUCTION

The present stage of development of space-system engineering is defined by significant growth of interest in the CubeSat nanosatellite development and launch. So according to the information source [www.nanosats.eu](http://www.nanosats.eu), in 2017, 295 nanosatellites were launched [1]. More than 37% of the launched nanosatellites were developed and manufactured by the universities conducting developments in the field of space technologies. Great interest in such class of spacecraft is due to the fact that their creation doesn't require significant financial expenses, allows researchers to apply technology of project-based learning and also to carry out flight tests of the onboard system elements and new technical solutions in space conditions before applying them in expensive space missions.

According to the common use of the CubeSat nanosatellite, the size of which is multiple of the size of 10x10x10 cm (1U format), single nanosatellites can't solve complex problems. The difficult missions characterized by simultaneous launch of several nanosatellites aimed at the solution of the same problem are developed for enlarging the field of their use. At the same time the concept of creation of the distributed spacecraft, representing a group of satellites flying in relatively short mutual distance from several tens of meters to hundreds of kilometers [2] can be realized.

Implementation of formation flight can be used for the solution of such problems as [3]:

- research of the payload features in the distributed spacecraft (for example, LUVEX project);
- research of geophysical fields in near-earth space (GRACE, SWARM, MMS, SCOPE projects);
- formation of antenna fields of the ultra-large aperture (XEUS, DARWIN, LISA projects);
- implementation of repeated multirange serial shooting of the Earth's surface or deep space (EO-1/LandSat-7, TanDEM-X, A-TrainTPF projects);
- spacecraft servicing in orbit, etc.

Some of the projects are listed in the Table 1.

TABLE I. PROJECTS ON CREATION OF DISTRIBUTED SPACECRAFT

Project	Performances		
	Application	Number of satellites	Distance between satellites, m
XEUS [4]	X-ray telescope	2	35
GRI [4]	Radio telescope	2	100
CanX-4/5 [5]	Demonstration of the nanosatellite formation flight	2	50-100
DTUsat [5]	Demonstration of the nanosatellite formation flight	2	50

For implementation of formation flight it is necessary to solve a number of problems:

- 1) providing the inter-satellite communication for collection and exchange of target and service information;
- 2) nanosatellite relative navigation and attitude determination for development of the control loop of nanosatellite formation;
- 3) formation and realization of the control strategy for maintenance of the set flight configuration of nanosatellite group in the Earth orbit motion.

For operation of the nanosatellite group in orbit providing relative navigation and attitude of the nanosatellites relative to each other or to the basic spacecraft is required. Because of limited information and energy resources, development of the effective algorithms for maintenance of formation and, if necessary, its reconfiguration is a complicated problem. However now there are various methods of this problem solution.

In most published works spacecraft relative navigation and attitude determination is based on processing of the video content received when shooting one spacecraft by means of the video camera installed on other spacecraft. [7, 8, 9]. A lack of this approach it is possible to denote rather large amount of necessary computation that does problematic the realization of the algorithm onboard the nanosatellite that generally has low calculating capacity. Besides that, realization of such methods is possible only at relatively small distance between objects.

Now the most routine methods are the methods based on use of measurements on the satellite radionavigation systems GLONASS/GPS [10, 11, 12]. Relative navigation determination on the basis of data from satellite radionavigation systems GLONASS/GPS requires installation a navigation receivers, on each nanosatellite. Considering high energy consumption of navigation receivers use of such approach for nanosatellites isn't always possible.

In [13] the method of determination of the relative motion parameters of two satellites using the NORAD data is described. A lack of this method is its small accuracy and also no operativity as it can't be realized independently onboard the satellite.

In [14] there was proposed a method for measuring distance using nanoLOC technology at distances up to 1000 m. The random error of measurements of this technology is within  $\pm 80$  cm, which was proved by experimental data.

In this research algorithms of relative navigation and attitude determination using measurements of ranges between reference points located on nanosatellites are offered. With the use of the measured ranges algorithms of nanosatellite relative navigation and attitude determination find relative location and mutual angular position of nanosatellites in group.

## II. RELATIVE NAVIGATION

### A. Mathematical statement of problem of the nanosatellite relative navigation determination at formation flight

Let's consider two nanosatellites flying near each other that have as a part of the onboard equipment the multiantenna system having possibility to measure range between the relevant antenna phase centers (reference points) located on each nanosatellite. It is supposed that the multiantenna system of the leading nanosatellite (leader) consists of two types of antennas, the transmit/receive antenna (Tx/Rx) and several receive antennas (Rx), the signal from that is used for calculation of ranges between the following nanosatellite (follower) and antennas of the leader (Fig. 1). The transmit/receive antenna (Tx/Rx) provides not only measurement of range between two nanosatellites, but also

service data exchange between nanosatellites. Radio engineering problems of the statement implementation have been proved in [15].

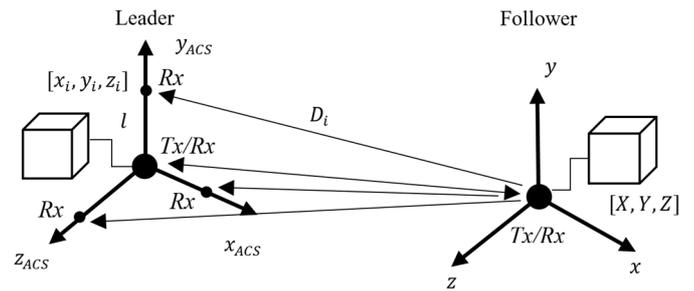


Fig. 1. The scheme of measurements of ranges between two nanosatellites

Measurement of range is made as follows, the follower send out a radio signal by the transmit/receive antenna, the leader pick up this signal by all visible antennas. As a result, there are several measurements of range from one reference point of the follower to several reference points of the leader. To remove the condition of antenna visibility/invisibility more receiving reference points can be located on the leader. In sight of the transmit/receive antenna of the follower there have to be at least 4 reference points of the leader.

In the problems of relative navigation determination based on ranging methods of radionavigation, an important role is played by precise measurement of range between the reference points located on nanosatellites. Due to rather small distance between nanosatellites, delays in receiving the direct signal are small, as a result the range sensor has to have high precision of range measurement.

For the relative navigation problem solution let's accept the following assumptions:

- as all measurements of relative navigation system are based on the group leader position, we will consider that the leader is three-axes stabilized and axes of the body frame of references of the leader coincide with axes of the orbital frame of references;
- it is considered that spatial position of the leader (the navigation receiver is installed onboard) is known;
- differences between the nanosatellite orbital frames of references are neglected that is fair at relative distance between satellites not exceeding 1 km (for example, at height of 500 km the mutual angular position of orbital frames of references doesn't exceed  $0,001^\circ$ );
- the nanosatellite is accepted as a radiotransparent body, the condition of visibility/invisibility isn't considered, at the same time the number of measurements of ranges "leader – follower" is accepted equal to 4;
- the onboard clock of nanosatellites are synchronized;
- the unfolding receiving and radiating antennas are considered to be absolutely rigid, the coordinates of the phase centers in the associated coordinate system are considered known.

Using coordinates of reference points of the leader and coordinates of center of mass of the follower, the equation of measurement of range between the follower and reference points of the leader may be written as:

$$D_n = \sqrt{\sum_{j=1}^3 (\eta_j^i - \eta)^2} \quad (1)$$

where  $\eta_i = (x_i, y_i, z_i)$  are coordinates of reference points of the leader in the body frame of references;  $i$  is number of a point, number of range ( $i=1 \dots n$ ) respectively;  $n$  is the number of the measured ranges between nanosatellites in group;  $\eta = (X, Y, Z)$  are coordinates of the follower center of mass.

Using coordinates of reference points of the leader  $\eta_i$  and taking into account the equation (1), the system of the measurement equations that solution will be follower coordinates  $\eta$  is written as:

$$\begin{cases} D_1 = \sqrt{\sum_{j=1}^3 (\eta_j^1 - \eta)^2} \\ D_2 = \sqrt{\sum_{j=1}^3 (\eta_j^2 - \eta)^2} \\ \dots \\ D_n = \sqrt{\sum_{j=1}^3 (\eta_j^n - \eta)^2} \end{cases} \quad (2)$$

### B. Numerical method of the nanosatellite relative navigation problem solution at formation flight

Let's analyze the possibility of solving the problem of relative navigation on board the leading nanosatellite using numerical methods of zero and first order, to minimize the objective function (3).

$$f(\eta) = (D_i - D_i^{meas})^2 \rightarrow \min \quad (3)$$

For comparison, the method of least squares [16], which requires knowledge of the initial approximation, and the differential evolution method [17], which involves specifying the region of initial parameters, were considered. Initial data of the problem are:

- 1) Set of range measurements  $D_i^{meas}$ .
- 2) Coordinates of reference points of the leader in the body frame of references  $\eta_i$ .

Required parameters are coordinates of the follower in the leader body frame of references  $\eta$ .

The differential evolution algorithm carries out multiparameter minimization of the objective function (3) in the set area of required parameters by the following steps [16]:

- 1) definition of the initial array  $X$  consisting of  $N$  vectors  $\eta$  ( $N$  in each iteration is constant and is one of parameters of the differential evolution algorithm), and each element of vector  $\eta$  is generated in a random way in the set area;

- 2) generation of a new array  $X_{new}$  of vectors  $\eta$ : for each vector  $\eta_i$  three various vectors  $\eta_1, \eta_2, \eta_3$  which indexes don't coincide with the index of vector  $\eta_i$  are obtained from the array  $X$  in a random way, and the so-called modified vector (mutantvector) is calculated by formula:  $\eta_m = \eta_1 + F \cdot (\eta_2 - \eta_3)$ , where  $F$  is the constant from the interval of values  $[0, 1]$  which is the parameter of the differential evolution algorithm;

- 3) with modified vector  $\eta_m$  the operation "crossover" is carried out. Under this operation some of  $\eta_m$  elements are replaced with the relevant elements from the initial vector  $\eta_i$  (each element is replaced with  $P$  probability that is also one of the parameters of the differential evolution algorithm);

- 4) if the received vector is better than the vector  $\eta_i$ , i.e. the value of the objective function decreases  $f(\eta_m) < f(\eta_i)$ , then in the new array  $X_{new}$  the vector  $\eta_i$  is replaced with a new vector  $\eta_i$  (is called trial vector), otherwise  $\eta_i$  remains;

- 5) process will continue until the value of the objective function  $f(\eta)$  reaches the minimum value.

The area of required parameters can be reduced binding measurement to the relevant reference point.

For the same initial data and a sample of measurements, the problem of relative navigation was solved using least squares method.

Error estimation of relative navigation problem solution by using both numerical methods is shown on Fig. 2.

Multiple solutions of the problem for various initial conditions show that the differential evolution method provides less methodological errors and is more convenient for implementation in on-board software for such a class of problems. Methodological error is increasing due to the deterioration of the observability conditions as the distance between satellites is increasing too.

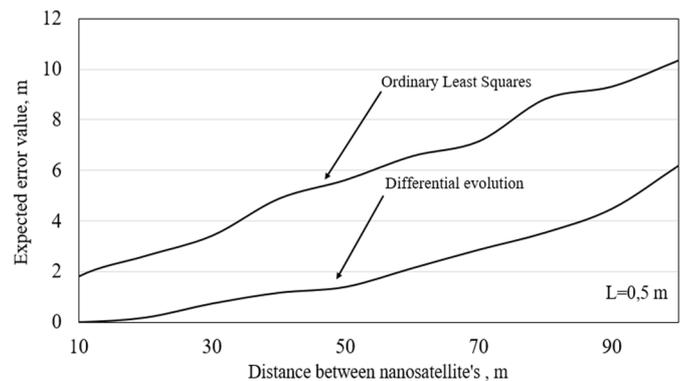


Fig. 2. Error estimation of relative navigation problem solution

The angle of view from the driven nanosatellite of the reference points on the leading nanosatellite versus distance is shown on Fig. 3.

It can be seen that, in the range of distances between nanosatellites within 80-100 m, the angle of visibility does not exceed  $0.5^\circ$ , which causes the termination of the convergence of both methods and the appearance of a methodical error.

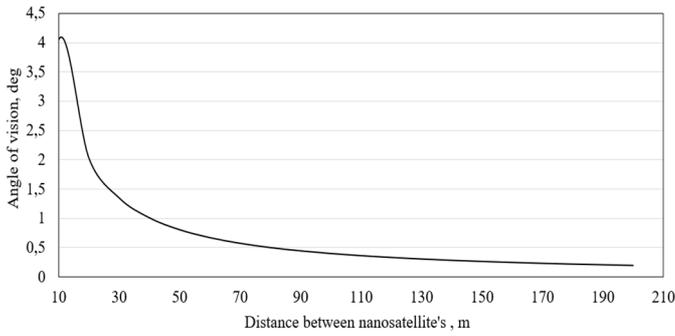


Fig. 3. The angle of visibility dependence from distance between nanosatellites

*C. The parametrical analysis of the nanosatellite relative navigation determination algorithm at formation flight in model problem*

At small sizes of the nanosatellite it is difficult structurally to realize  $l$  distance more than 0,5 m between the nanosatellite center of mass and reference point, further it is called conditional base of measurements (it is possible to provide with extensible antennas, Fig. 4). The size of conditional base of measurements limits the maximum appropriate distance between spacecraft in group as at increase in distance between nanosatellites the observability matrix determinant tends to zero and the system degenerates.

In model problem the research of influence of conditional base of measurements ( $l$ ), distance between nanosatellites ( $r$ ), error of range measurement ( $\Delta$ ) on error of the relative navigation problem solution was conducted. By results of research the algorithm field of applicability has been formed.

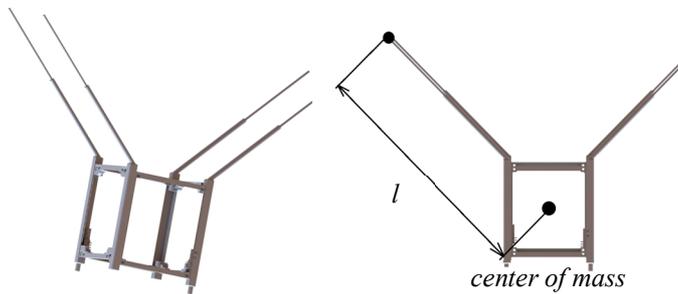


Fig. 4. Nanosatellite with extensible antennas

When simulation the following assumptions have been made:

- the research of influence of conditional base of measurements was carried out for values from 0,1 m to 0,5 m, in increments of 0,1 m;
- the distance between nanosatellites doesn't exceed 500 m.

To research the influence of distance between nanosatellites on the method error of the relative navigation problem solution, distance between nanosatellites was split into intervals, multiple of 10 m. For receiving estimates of method error expected value

on each interval the sample of the follower coordinates of 400 realizations was generated.

The analysis of error of the relative navigation problem solution was carried out in two steps.

At the first step of simulation influence of distance between nanosatellites and influence of size of conditional base of measurements on algorithm method error expected value was estimated, in addition the range measurement error was accepted equal to zero. Results of simulation are presented in Fig. 5.

In Fig. 5 and 6 dependences of expected value of method error of the follower navigation problem solution on distance between two nanosatellites at various values of base are shown.

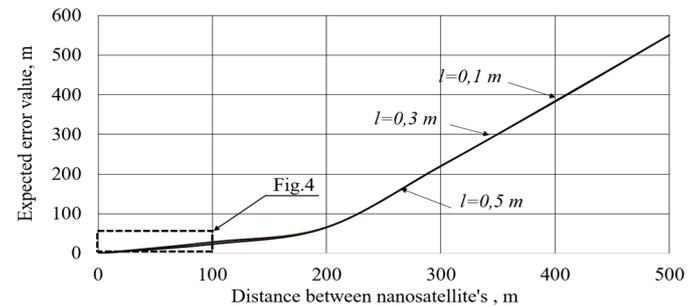


Fig. 5. Expected value of the method error of the navigation problem solution

Fig. 5 demonstrates that after 200 m error of the navigation problem solution on distance between nanosatellites has linear character. As can be seen from Fig. 5 and Fig. 6 this algorithm works with the acceptable accuracy (no more than 15 m) at distance up to 100 m. The analysis of the existing missions for formation creation has shown that the distance between spacecraft in formation is within 100 m (table 1) that does possible the algorithm use for relative navigation determination at distances up to 100 m between nanosatellites.

Fig. 6 shows that as conditional base of measurements increases the error of the problem solution decreases.

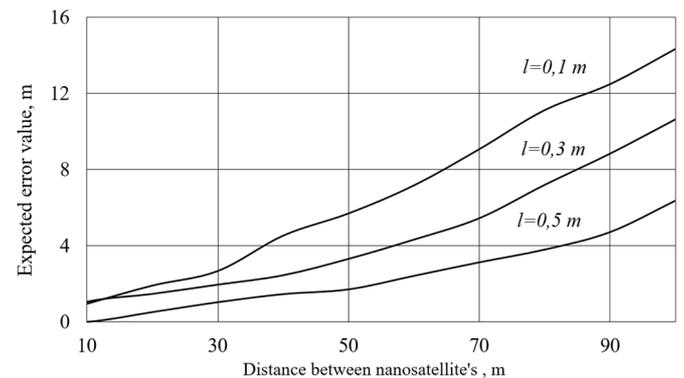


Fig. 6. Expected value of the method error of the navigation problem solution

For obtaining analytical dependence of method error expected value on distance between nanosatellites and on conditional base of measurements we approximate results of numerical simulation by polynom of the second order (in Fig. 7 by dotted line):

$$M_{\varepsilon}(r, l) = A(l)r^2 + B(l)r + C(l) \quad (4)$$

where  $M_{\varepsilon}$  is expected value of method error of the follower relative navigation problem solution,  $r$  is distance between nanosatellites;  $A(l)$ ,  $B(l)$ ,  $C(l)$  are the coefficients of the second order polynom equation that are functions of conditional base of measurements.

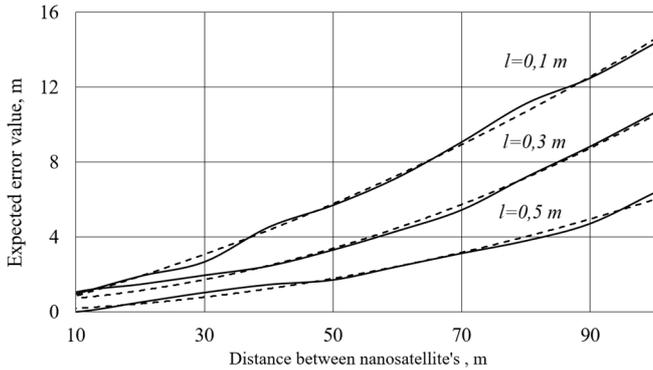


Fig. 7. Approximation of expected value of method error of the navigation problem solution

For receiving the analytical equation of dependence of  $A, B, C$  coefficients on conditional base of measurements, we approximate results of numerical simulation by the following polynoms:

$$\begin{aligned} A(l) &= \sum_{i=1}^5 a_i l^{5-i} \\ B(l) &= \sum_{i=1}^7 b_i l^{7-i} \\ C(l) &= \sum_{i=1}^5 c_i l^{5-i} \end{aligned} \quad (5)$$

Numerical values of approximation coefficients  $a_i$ ,  $b_i$ ,  $c_i$  depend on sample volume (for example, for sample of 400 realizations, Table 2). Using the equations (4) and (5) it is possible to estimate the expected value of algorithm method error, the influence of distance between nanosatellites and conditional base of measurements.

TABLE II. EQUATION (4) COEFFICIENTS

i	1	2	3	4	5	6	7
$a_i$	-0.011	0.027	-0.024	0.008	0.0005		
$b_i$	-4.833	16.169	-19.978	10.45	-1.379	0.564	0.149
$c_i$	-6.884	18.135	-16.151	5.088	0.294		

At the second step influence of error of range measurement ( $\Delta$ ) on the relative navigation problem solution was estimated.

Modern methods of radio signal processing allow to measure ranges with error less than 0,5 m [18] that is why in research it was accepted that the error of measurement of range between nanosatellites didn't exceed 0,5 m. In such a case the relative navigation problem was solved for three values of conditional base of measurements: 0,1; 0,3 and 0,5 m.

In Fig. 8 the dependence of expected value of error of the follower relative navigation problem solution depending on distance between nanosatellites at various errors of range measurement is shown.

Fig. 8 represents that the range measurement error exerts slight impact on the relative navigation problem solution.

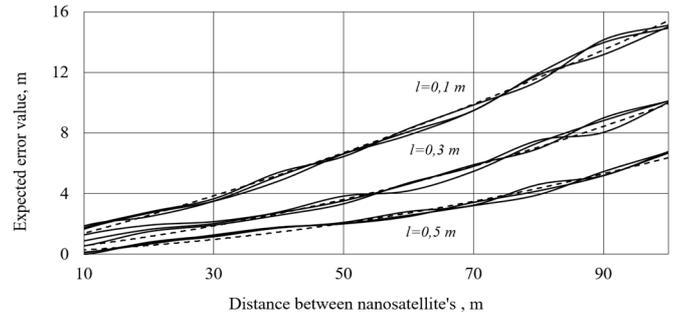


Fig. 8. Expected value of the relative navigation problem solution error

### III. RELATIVE ATTITUDE

#### A. Mathematical statement of problem of the nanosatellite relative attitude determination at formation flight

In this case let's understand as relative attitude the mutual angular position between the body frames of references of the leader and the follower.

We will consider two nanosatellites flying near each other that have the multiantenna system shown in Fig. 9 as a part of the onboard equipment. The main difference of this system from considered above (Fig. 1) is that the follower has not less than 4 transmit/receive antennas (Tx/Rx).

Range measurement is made as follows, the follower send out a radio signal by all reference points, the leader pick up this signal by the receiving reference points. As a result we have measurements of range between different reference points (Fig. 9).

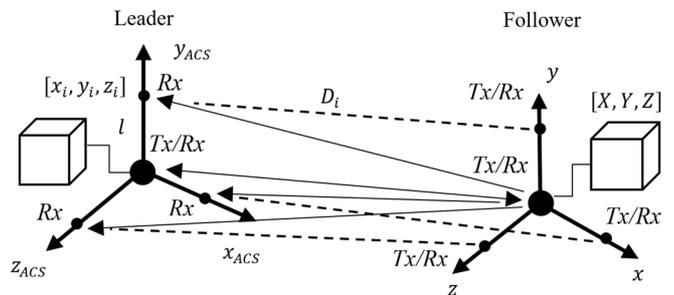


Fig. 9. The scheme of measurements of range to solve the relative attitude problem

The assumptions accepted for the relative attitude problem solution:

- position of the follower center of mass in the body frame of references of the leader is known (result of the relative navigation problem solution);
- other assumptions are similar to the assumptions accepted above for the relative navigation problem solution.

Similar to the equation (1), using coordinates of reference points of the leader and the follower, we will write down the equation of measurement of range between reference points on the leader and reference points of the follower:

$$D_{ik} = \sqrt{\sum_{j=1}^3 (\eta_j^i - \eta_j^k(\alpha, \beta, \gamma))^2} \quad (6)$$

where  $\eta_j^i = (x_i, y_i, z_i)$  are coordinates of reference points of the leader in its own body frame of references;  $i, k$  are numbers of reference points of the leader and the follower respectively;  $\eta_j^k(\alpha, \beta, \gamma)$  are coordinates of reference points of the follower in the body frame of references of the leader that are:

$$\eta^k(\alpha, \beta, \gamma) = M \times \xi^k + \eta^{LM} \quad (7)$$

where  $M$  is transfer matrix from the follower body frame of references to the leader body frame of references,  $\xi^k$  are coordinates of  $k$  reference point of the follower in its own body frame of references,  $\eta^{LM}$  are coordinates of the follower center of mass.

$$M = \begin{bmatrix} c(\beta)c(\alpha) & -s(\alpha) & s(\beta)c(\alpha) \\ s(\beta)s(\gamma) + s(\alpha)c(\beta)c(\gamma) & c(\alpha)c(\gamma) & s(\beta)c(\gamma)s(\alpha) - s(\gamma)c(\beta) \\ s(\alpha)c(\beta)s(\gamma) - s(\gamma)c(\gamma) & c(\alpha)s(\gamma) & c(\beta)c(\gamma) + s(\alpha)s(\beta)s(\gamma) \end{bmatrix} \quad (8)$$

where  $c$  is  $\cos$ ;  $s$  is  $\sin$ ;  $\alpha, \beta, \gamma$  are the angles of relative spatial attitude describing turn of the follower body frame of reference about the leader body frame of reference Fig. 10.

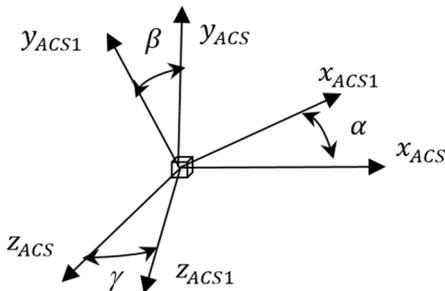


Fig. 10. Relative attitude angles

Using coordinates of reference points of the leader  $\eta^i$ , and also coordinates of the follower center of mass, taking into account the equation (6), let's write down the system of the equations of measurements solution will be follower relative attitude angles  $\alpha, \beta, \gamma$

### B. Numerical method of the nanosatellite relative attitude problem solution at formation flight

The relative attitude problem solution is the result of minimization of the objective function with the use of the differential evolution algorithm.

The problem initial data are:

- 1) Set of range measurements "the leader - the follower"  $D_{ik}^{meas}$ ;
- 2) Coordinates of reference points of the leader in its own body frame of references of the leader,  $\eta_i$ ;
- 3) Coordinates of the follower center of mass in the body frame of references of the leader,  $\eta^{LM}$ .

The angles of follower relative attitude  $\alpha, \beta, \gamma$  are required.

Sequence of computations is the following:

- 1) Formation of measurements of ranges between the reference points located on the follower and the leader  $D_{ik}^{meas}$ ;
- 2) Minimization of the objective function (9) with the use of the differential evolution algorithm.

$$f(\alpha, \beta, \gamma) = (D_{ik} - D_{ik}^{meas})^2 \rightarrow \min \quad (9)$$

### C. The parametrical analysis of the nanosatellite relative attitude determination algorithm at formation flight in model problem

In model problem the research of influence of conditional base of measurements ( $l$ ), distance between nanosatellites ( $r$ ), error of range measurement ( $\Delta$ ) on error of the relative attitude problem solution was conducted. By results of research the algorithm field of applicability has been formed.

When simulation the following assumptions have been made:

- research of influence of conditional base of measurements was carried out for values from 0,1 m to 0,5 m, in increments of 0,1 m;
- distance between nanosatellites doesn't exceed 100 m.

The research of influence of distance between nanosatellites on method error of the relative attitude problem solution was conducted similar to the relative navigation problem.

At the first step the expected value of the algorithm method error was estimated, notice that the error of range measurement  $\Delta$  is equal to zero.

In Fig. 11 the dependence of expected value of method error of the follower relative attitude problem solution on conditional base of measurements is shown.

Fig. 11 shows that as conditional base of measurements increases the error of the relative attitude angle determination decreases.

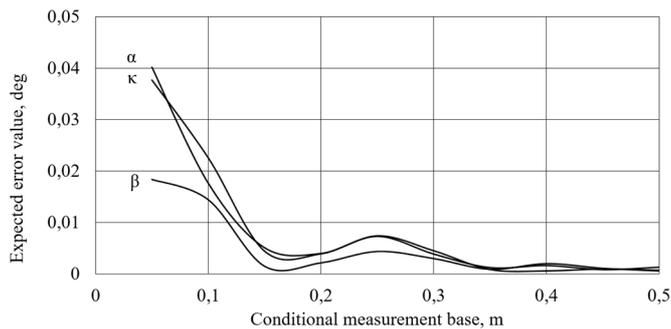


Fig. 11. Expected value of method error of the relative attitude problem solution

At the second step influence of error of measurement of range between nanosatellite reference points was estimated.

We accept that the error of measurement of range between spacecraft doesn't exceed 0,5 m, in such a case the relative navigation problem was solved for conditional base of measurements of 0,1, 0,3 and 0,5 m. The result is given in Fig. 12.

Follows from Fig. 12 at increase in range measurement error the error of the relative attitude angle determination grows, however in the worst case ( $l=0,1$  of m and  $\Delta=0,5$  m) the error of the relative attitude problem solution is within  $10^\circ$ .

Preliminary estimates show that an increase in accuracy to  $1^\circ$  can be achieved using a magnetometer.

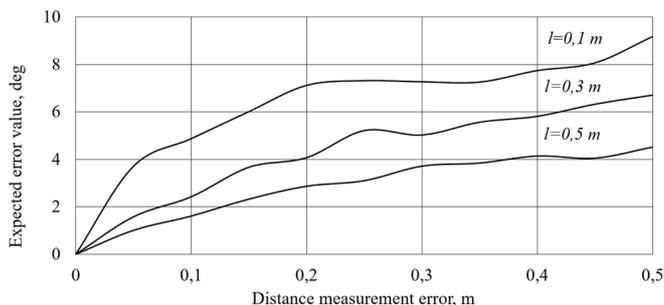


Fig. 12. Expected value of error of the follower relative attitude angle determination depending on the range measurement error

#### IV. CONCLUSION

In the research nanosatellite relative navigation and attitude determination algorithms at formation flight on the basis of range measurements are developed and investigated. Estimates of error of the relative navigation and attitude problem solution depending on the range measurement error, distances between nanosatellites and conditional base of measurements are received.

Apparently from simulation results, the nanosatellite relative navigation problem solution algorithm is reasonably applied at distance between nanosatellites up to 100 m that it is possible to carry to disadvantages of this algorithm, the error of the navigation problem solution doesn't exceed 15 m.

Using the differential evolution method for relative navigation problem solution does not require high-precision a priori values of the center of mass coordinates of the driven nanosatellite, which refers to the advantages of this approach.

The nanosatellite relative attitude problem solution algorithm as well as the relative navigation algorithm is applied at distance between nanosatellites up to 100 m, error of the relative attitude angle determination doesn't exceed  $10^\circ$ .

The obtained results allow to create technical requirements to the relative navigation and attitude system.

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