PASSIVE THREE-AXIS STABILIZATION OF A NANOSATELLITE IN LOW-ALTITUDE ORBITS: FEASIBILITY STUDY*

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***Abstract*—The possibility of implementing passive three-axis stabilization of a nanosatellite of the CubeSat format in low circular orbits is shown for the case when the aerodynamic moment is determining the motion of the nanosatellite relative to the center of mass, which ensures the stabilization of the longitudinal axis of the nanosatellite, while the stabilization of the transverse axes of the nanosatellite is due to the gravitational moment. The nomograms that allow selecting the main design are obtained.**

***Keywords*—nanosatellite; three-axis stabilization; attack angle; roll angle; design parameters**

I. INTRODUCTION

To ensure a given orientation of nanosatellites in space, passive or combined (passive in combination with active) stabilization systems are generally used. They do not require or require small expenditure of the working body and the energy stored on board.

As you know, the design conditions such as uncontrolled motion of satellites can only be ensured at the design stage by selecting its design and ballistic parameters, as well as specifying limits of the angular velocity generated by the separation system, or in combined system of stabilization at the end of operation of the preliminary damping system.

Most nanosatellites are launched into low circular orbits, at such heights gravitational and aerodynamic moments dominate and to stabilize the angular position it is advisable to use both of the moments.

In this paper, the task of ensuring the stabilization of the nanosatellite is solved taking into account the uncertainty in the characteristics of the initial angular motion of the nanosatellite when launching into orbit or at the end of operation of the preliminary damping system (in contrast to the well-known works in which this problem is solved in deterministic statement, for example [1]). This work is a continuation of the authors' research on the development and practical application of methods of passive stabilization of nanosatellites [2] - [10].

In [2], probabilistic models of the initial conditions of the nanosatellite angular motion were formed after the associated launch from the orbital stage of the "Soyuz" carrier rocket, which makes uncontrolled motion after separation of the main payload [3]. In [4] the uncontrolled angular motion of a nanosatellite around its center of mass after separation from stabilized platform was considered. The analytical probability density function and the cumulative distribution function of the maximum nanosatellite angle of attack for Rayleigh and uniform distributions laws of the initial transverse angular velocity value are obtained. In [5], [6] the task of ensuring the aerodynamic stabilization of a nanosatellite is considered. The formulas for choosing the design parameters of aerodynamically stabilized nanosatellite (statical stability factor, length of nanosatellite, longitudinal moment of inertia) were obtained on the basis of these analytical distribution laws of the maximum angle of attack. The calculated design parameters provide the deviation value of the longitudinal axis of a nanosatellite from velocity vector less than acceptable with a given probability at a given altitude of its separation and at the known errors of initial angular separation velocity. The Eurasian patent of single-axis aerodynamic stabilization method for CubeSat standard nanosatellite [7] was obtained according to the results of research. This method of single-axis aerodynamic stabilization was used in 2 projects of Samara National Research University. The first of them is creation of the nanosatellite SamSat-218D with passive aerodynamic stabilization system [8]. The second of them is creation of the aerodynamically stabilized nanosatellite SamSat-QB50 with deployable stabilizer [9] included in the international QB50 project. In [10], the possibility of three-axis gravitational-aerodynamic stabilization of the nanosatellite was substantiated for the case when the motion of the nanosatellite with respect to the center of mass is determined by the gravitational moment, while the aerodynamic moment helps stabilize the motion.

In this paper, we propose an approach to ensuring three-axis stabilization of the nanosatellite in low circular orbits for the case when the aerodynamic moment is determining the motion of the nanosatellite relative to the center of mass, which ensures the stabilization of the longitudinal axis of the nanosatellite, while the stabilization of the transverse axes of the nanosatellite is due to the gravitational moment.

II. ANALYSIS OF ANGULAR MOTION

An analysis of the motion of a dynamically symmetric nanosatellite relative to its center of mass under the action aerodynamic and gravitational moments was carried out in [4]. Analytical cumulative distribution function and probability density function of the maximum angle of attack of the nanosatellite for Rayleigh and uniform distributions laws of the initial transverse angular velocity value are obtained, on the basis of which the of single-axis aerodynamic stabilization of the nanosatellite of the CubeSat format is solved.

Let us analyze the possibility of stabilizing the transverse axes of the nanosatellite due to the gravitational moment, assuming that the longitudinal axis of the nanosatellite is stabilized relative to the velocity vector of the center of mass (the angle of attack is small). Then, the oscillations of the transverse axes in a plane perpendicular to the velocity vector of the center of mass can be described approximately by an equation of the form:

$$\ddot{\delta} = \frac{3\mu}{2(R_E + H)^3}\left(\frac{J_z - J_y}{J_x}\right)\sin 2\delta\,, \qquad (1)$$

where J_x is the principal moment of inertia about the longitudinal axis; J_y, J_z are the principal moments of inertia of nanosatellite; δ is the roll angle (the angle of deviation of the transverse axis Oz, for which the value of the principal moment of inertia J_z takes a value that satisfies the condition $J_x < J_z < J_y$); μ is the Earth's gravitational parameter; R_E is the radius of the spherical Earth; H is the flight altitude.

The altitude of the circular orbit because of atmospheric drag changes very slowly and when considering the angular motion of the nanosatellite on one or more turns, we can assume that $H = const$. In this case, the system (1) has an integral of energy:

$$\dot{\delta}^2/2 + d\cos 2\delta = E_0\,, \qquad (2)$$

where $E_0 = d\cos 2\delta_0 + \frac{\omega_{x0}^2}{2}$ is determined by the initial conditions of angular motion; $\omega_{x0} = \dot{\delta}_0$ is the initial longitudinal angular velocity; $d = \frac{3\mu}{4(R_E + H)^3}\left(\frac{J_z - J_y}{J_x}\right)$.

Then the value of the maximum roll angle δ for oscillations in the range $[0, \pi/2]$ can be found from the energy integral (2) for $\dot{\delta} = 0$:

$$\delta_{\max} = \frac{1}{2}\arccos\left(\frac{E_0}{d}\right). \qquad (3)$$

From the values included in (3), the greatest scatter of values is the magnitude of the longitudinal angular velocity ω_{x0}. Then, neglecting the scatter of other values, calculating the distribution of the function $\delta_{\max}$ with respect to the distribution of the argument ω_{x0} in accordance with [11], we obtain analytical expressions for the cumulative distribution function of the value of the maximum roll angle $\delta_{\max}$ for two variants of the laws of distribution of the magnitude modulus ω_{x0}.

Let the value ω_{x0} have a normal distribution with mean zero and standard deviation σ. Then the cumulative distribution function of the modulus of ω_{x0} is given by:

$$F(\omega_{x0}) = 2\Phi_0\left(\frac{\omega_{x0}}{\sigma}\right), \qquad (4)$$

and the cumulative distribution function of the value of the maximum roll angle $\delta_{\max}$ takes the form:

$$F(\delta_{\max}) = 2\Phi_0\left(\frac{\sqrt{2d(\cos 2\delta_{\max} - \cos 2\delta_0)}}{\sigma}\right), \qquad (5)$$

where $\Phi_0(t) = \frac{1}{\sqrt{2\pi}}\int_0^t e^{-t^2/2}dt$ is the normal distribution function.

Let the modulus of ω_{x0} have a distribution according to the uniform law in the range $[0, \omega_{x0\max}]$:

$$F(\omega_{x0}) = \begin{cases} \frac{1}{\omega_{x0\max}}, & \omega_{x0} \in [0, \omega_{x0\max}] \\ 0, & \omega_{x0} \notin [0, \omega_{x0\max}] \end{cases}, \qquad (6)$$

then the cumulative distribution function of the modulus of $\delta_{\max}$ is given by:

$$F(\delta_{\max}) = \frac{\sqrt{2d(\cos 2\delta_{\max} - \cos 2\delta_0)}}{\omega_{x0}}. \qquad (7)$$

III. SESLECTION OF DESIGN PARAMETERS

Passive three-axis stabilization of the nanosatellite of the CubeSat format in low circular orbits is provided by the selection of its design parameters.

Stabilization of the longitudinal axis of the nanosatellite is due to the selection of its design parameters (statical stability factor, geometric dimensions, principal moment of inertia J_y), providing for the motion in low orbits, the deviation of the longitudinal axis of the nanosatellite from the velocity vector of the center of mass is less than the acceptable at the required altitude with a given probability for given errors of the transverse angular velocity from the separation system, or in combined system of stabilization at the end of operation of the preliminary damping system, in accordance with [5] - [7].

Below we obtain the formulas for the selection of magnitudes of the principal moments of inertia, which ensure stabilization of the transverse axes of the nanosatellite.

Setting p^* as the probability of realizing the allowable value of the maximum roll angle δ_{max} and solving expressions (5), (7) with respect to the design parameters combined in the constructive parameter $d_k = \frac{J_y - J_z}{J_x}$ of the nanosatellite, we obtain a requirement for its magnitude. In order for the maximum roll angle δ_{max} (the angle of deviation of the transverse axis of the nanosatellite from the plane of flight) to be less than the permissible value with a probability not less than p^* for the given variations in the longitudinal angular velocity generated by the separation system, or in the combined system of stabilization at the end of operation of the preliminary damping system, it is necessary to fulfill the following condition for the design parameter of the nanosatellite:

- if the value ω_{x0} has the normal distribution with mean zero, and the magnitude module ω_{x0} has the cumulative distribution function (4):

$$d_k = \frac{J_y - J_z}{J_x} \geq \frac{4(R_E + H)^3}{3\mu} \frac{\sigma^2 \left(t^*\right)^2}{2(\cos 2\delta_0 - \cos 2\delta_{max})}, \quad (8)$$

where t^* is the argument of the normal distribution function for the given probability: $\Phi_0\left(t^*\right) = p^* / 2$;

- in the case of the distribution of the modulus of the initial longitudinal angular velocity ω_{x0} according to the uniform law in the range $[\,0,\, \omega_{x0\,max}\,]$:

$$d_k = \frac{J_y - J_z}{J_x} \geq \frac{4(R_E + H)^3}{3\mu} \frac{(\omega_{x0\,max} p^*)^2}{2(\cos 2\delta_0 - \cos 2\delta_{max})}. \quad (9)$$

Using the obtained expressions (8), (9), it is possible to construct nomograms for estimating the possibility of providing the required value of the design parameter d_k. For example, Fig. 1, on the right, shows the dependencies of the required design parameter of the nanosatellite d_k on the permissible roll angle δ^*_{max} and on the value of the right boundary of the initial longitudinal angular velocity $\omega_{x0\,max}$ (the initial longitudinal angular velocity is distributed according to a uniform law) calculated for the initial angle of roll $\delta_0 = 0$, the probability $p^* = 0.95$ and altitude of the flight $H = 380$ km; on the left, we can see the values of the design parameter of the nanosatellite with different values of the intermediate principal moment of inertia J_z, depending on the least principal moment of inertia J_x for the greatest principal moment of inertia $J_y = 0.025\ \text{kg} \cdot \text{m}^2$.

The nomograms can be used both to select the design parameters of the nanosatellite and to specify the requirements for the deviation of the initial longitudinal angular velocity.

IV. CONCLUSIONS

Thus, the possibility of implementing passive three-axis stabilization of a nanosatellite of the CubeSat format in low circular orbits has been shown. Formulas for the selection of the design parameters of the nanosatellite (statical stability factor, geometric dimensions, the principal moments of inertia) have been obtained that ensure three-axis stabilization (the longitudinal axis relative to the velocity vector of the center of mass, transverse axes relative to the plane of flight) for the motion in low orbits.

The nomograms have been constructed which allow simple and obvious selection of the main design parameters of the nanosatellite. Verification calculations on the spatial model of motion of the nanosatellite relative to the center of mass have confirmed the effectiveness of the decisions taken.

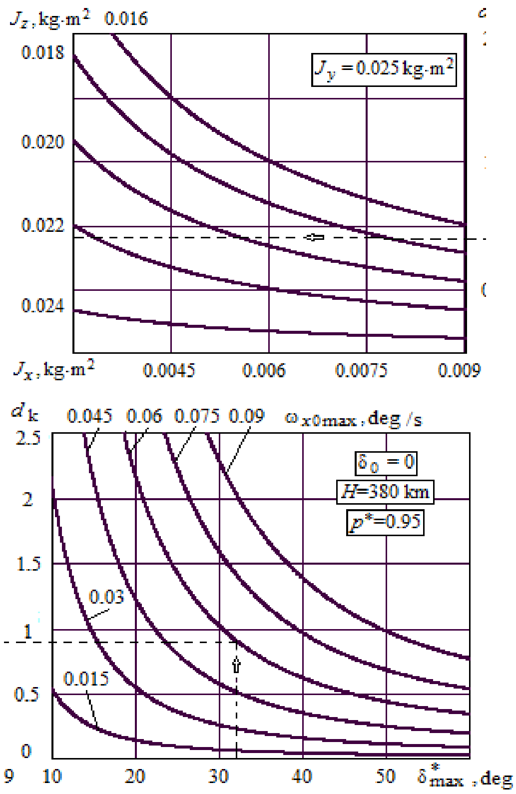

Fig. 1. Nomogram for selecting the design parameter of a nanosatellite.

ACKNOWLEDGMENT

The study was carried out with the financial support of the Ministry of Education and Science of the Russian Federation.

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