INVESTIGATION OF THE POSSIBILITY OF DETERMINING THE INERTIAL CHARACTERISTICS AND THE ANGULAR VELOCITY VECTOR OF CHAOTICALLY ROTATING SPACE DEBRIS OBJECT USING A NANOSATELLITE*

I.V. Belokonov
Space research department
.Samara University, Russia

I.A. Lomaka
Space research department
Samara University, Russia
igorlomaka63@gmail.com

***Abstract*—In this paper, the problem of identifying inertial characteristics of a space debris object using measuring instruments installed on board a nanosatellite is considered. The work assesses the possible use of the SamSat nanosatellite platform, which was developed in Samara University, using the onboard MPU-9250 chip with an integrated three-axis angular velocity sensor. The upper stage of the Kosmos-3M rocket is considered as an example of a large-sized object of space debris.**

***Keywords*—nanosatellite platform; angular velocity sensor; parametesr identification; inertial characteristics; space debris**

I. INTRODUCTION

At present, the problem of space debris is becoming more topical, as the number of spacecraft launches into high orbits is increasing. Especially large objects, for example, the upper stages of carrier rockets, which were not de-orbited after use [1] are dangerous for space missions. The problem of removing objects of space debris is aggravated due to unknown inertial characteristics, a complicated unknown rotation on several axes at high angular speeds, which makes it difficult to capture and transport space debris. The lack of reliable information about the angular velocities of the object and its inertial characteristics requires the development of a special methodology for their determination, which is the goal of this work. To solve this problem, it is proposed to use the nanosatellite as an auxiliary instrument to the space complex for capturing and de-orbiting space debris.

Today, there are a lot of works estimating the inertial characteristics of space debris. Thus, in [2] the inertia tensor and other parameters of the object are estimated using the extended Kalman filter based on the results of processing stereo photographs. In [3], stereocameras are also used to estimate moments of inertia, but the authors propose their own algorithm that allows the moments of inertia of space debris to be determined with an error of ± 8%. Modification of this algorithm as shown in [4] allows obtaining an error of identification of moments of inertia of the order of 0.001 kg m^2.

In addition to real-time algorithms using the Kalman filter, post-processing algorithms based on the method of least squares method are widely used (the Newton-Gauss algorithm and the Levenberg-Marquardt algorithm) [5] - [7]. Such algorithms are often used for in-flight estimation of spacecraft inertial characteristics and for its rotational motion reconstruction, but can be adapted for identification problem of space debris. For example, in [5], the inertial inertia tensor of Yamal satellite is identified by telemetric information. A similar approach was used in [6] for the international space station, as well as for in-flight determination of the Cassini's spacecraft [7], while the accuracy of the determination was ± 10%.

This article suggests an approach for identifying the parameters of space debris using nanosatellites (NS), which consists in separating the NS from the spacecraft (SC) - the garbage tow truck, and then attached to the space debris object (the attachment method is not considered in the article), then using on-board measuring unit determines dynamic and inertial characteristics, thus fulfilling the role of a remote measuring instrument (this approach was called the contact method). This approach has the advantage over, for example, [2], because it allows you to get rid of the problems associated with the use of optical or infrared cameras, planning the time of photographs. In addition, with the help of a nanosatellite, the task of establishing a physical connection between the spacecraft and the space debris can also be solved. To illustrate the solution of the problem, this article considers the second stage of the Kosmos-3M rocket.

Before 2012, more than 400 launches of this rocket for high-circular orbits were made. About 248 upper stages that make up 6% of the mass of all space debris make an uncontrollable flight in orbits at heights of 750-950 km, which is why there is a danger of their mutual collision and an avalanche increase in the number of fragments [1]. The design parameters of the dry construction of the second stage of the Kosmos-3M, which can be used as the initial approximation, are given in Table 1.

TABLE I. THE DESIGN PARAMETERS OF THE DRY CONSTRUCTION OF THE SECOND STAGE OF THE KOSMOS-3M

Parameter	Value
Mass (kg)	1720
Length (m)	4.2
Diameter (m)	2.4
Inertia moment Ix	1238
Inertia moment Iy	4200
Inertia moment Iz	4200

II. TECHNOLOGY OF SPACE DEBRIS REMOVAL

The proposed space debris removal technology includes the following steps:

- Separation of the nanosatellite from the main spacecraft.
- Measurement of the components of the vector of the angular velocity of the space debris object with the help of the onboard inertial measuring complex of the nanosatellite.
- Solving the problem of identifying the inertial characteristics of space debris.
- Transferring the solution results of space debris parameters on the main spacecraft.
- Use of information received by the spacecraft - for identification of the model of movement of a space debris object and formation of a strategy for its capture and removal.

In the study of the problem of identifying inertial characteristics, the following assumptions were made. First, the mechanical connection of the nanosatellite - space debris is absolutely rigid. Second, in the on-board algorithm for solving the problem, the model of motion of the space debris object relative to the center of mass is used, without taking into account the perturbing moments. The first assumption is made in with the fact that at this stage of the research the way for attaching a nanosatellite to an object of space debris is not determined. The second assumption is made in consequence of a short measurement time during which the perturbations rejected in the models will not make an appreciable contribution to the angular velocity measurements and will not affect the solution of the problem.

For the numerical solution of the identification problem the Gauss-Newton method is used. This method allows us to find the minimum of the objective function, which is the integral of the sum of squares of errors between the measured projections of angular velocity values and their model values. For successful solution of the identification problem, it is necessary to know the initial vector of estimated parameters. Therefore, one of the tasks of this paper is to determine the area of initial approximations.

III. PROBLE FORMULATION

By using angular velocity measurements $\zeta_{xi}, \zeta_{yi,} \zeta_{zi} (i = 1,2,3...,N)$ determine the mass inertial characteristics of the upper stage of the carrier rocket, the rotational motion of which is described by the dynamic Euler equations (1):

$$\begin{aligned} \dot{\Omega}_{x1} &= -k_x \Omega_{y1} \Omega_{z1} \\ \dot{\Omega}_{y1} &= k_y \Omega_{x1} \Omega_{z1} \\ \dot{\Omega}_{z1} &= -k_z \Omega_{x1} \Omega_{y1} \end{aligned} \quad (1)$$

where k_x, k_y, k_z – dimensionless inertia coefficients, which are calculated by the following equations $k_x = \frac{I_z - I_y}{I_x}$, $k_y = \frac{I_z - I_x}{I_y}$, $k_z = \frac{I_y - I_x}{I_z}$. Due to the lack of information on the current mass of the stage (the residual mass of the fuel is unknown) only dimensionless inertia coefficients can be found (any two coefficients of three, the third coefficient is already calculated unambiguously). The measurements are the projections of the angular velocity of the step on the axes of the coupled nanosatellite coordinate system $\Omega_x(t)$, $\Omega_y(t)$, $\Omega_z(t)$ which are related to the angular velocity projections of the step onto the main central axis of the step by the relation:

$$\vec{\Omega} = A\,\vec{\Omega}_1 \quad (2)$$

where – rotation matrix from the main inertia axes of the stage to the measuring axes of the three-axis angular velocity sensor of the nanosatellite, $\varphi_1, \varphi_2, \varphi_3$ – the angles between the main axes of the stage and the axes of the angular velocity sensor mounted on the nanosatellite Since the orientation of the measuring axes is fixed before the fixation of the nanosatellite on the stage, the angles $\varphi_1, \varphi_2, \varphi_3$. Then the expanded vector of the estimated parameters takes the form:

$$b^T = \left(\Omega_{x1}^0, \Omega_{y1}^0, \Omega_{z1}^0; k_y,\ k_z;\ \varphi_1,\ \varphi_2,\ \varphi_3 \right) \quad (3)$$

The desired vector of the parameters being evaluated must satisfy the minimum of the objective function:

$$J(b) = \frac{1}{2} \sum_{\alpha = x,y,z} \sum_{i=1}^{N} \left[\zeta_{\alpha i} - \Omega_\alpha (b,\ t_i) \right]^2 \quad (4)$$

IV. PROBLEM SOLUTION ALGORITHM

The problem is solved by finding the minimum of the objective function $J(b)$ using the Gauss-Newton method. The solution is in accordance with the algorithm [7]:

$$b^{k+1} = b^k - \left[G^k \right]^{-1} \nabla J^k \quad (5)$$

$$\nabla J^k = - \sum_{\alpha = x,y,z} \sum_{i=1}^{N} \left(\zeta_{\alpha i} - \Omega_\alpha{}^k \right) \nabla \Omega_\alpha{}^k \quad (6)$$

$$G^k = \sum_{\alpha=x,y,z} \sum_{i=1}^{N} \left[\nabla\Omega_\alpha{}^k \nabla^T \Omega_\alpha{}^k \right] \quad (7)$$

$$\nabla^T \Omega_\alpha{}^k = \left(\frac{\partial \Omega_\alpha{}^k}{\partial b_1}, \frac{\partial \Omega_\alpha{}^k}{\partial b_2}, \ldots, \frac{\partial \Omega_\alpha{}^k}{\partial b_8} \right) \quad (8)$$

The algorithm approximates the measurements of the angular velocities $\Omega_x(t)$, $\Omega_y(t)$, $\Omega_z(t)$ by solving the system of differential equations (1) and requires knowledge of the initial approximation area for the sought estimable vector within which the convergence of the method is ensured. A similar numerical algorithm was used in [7] to determine the inertial characteristics of the spacecraft for another formulation of the problem.

V. Determinination of Initial Value Area

Let us consider the technique of constructing the area of initial approximations for the estimated parameter vector, within which the Gauss-Newton method converges, as applied to the second stage of the Kosmos-3M. According to [8], the orbital stage of the Kosmos-3M rocket can rotate up to speed of 72 deg/s, and the angular velocity around the longitudinal axis is much less than the other two.

A. *The influence of perturbing moments on the angular tmotion of a space debris object*

A study is made of the effect of the earlier assumption that the influence of perturbing moments on the change in motion relative to the center of mass of the orbital step is negligible on short measurement intervals. Since for orbits with an altitude of 750-950 km the aerodynamic torque is much smaller than the gravitational torque, it is sufficient to estimate the influence of only the gravitational torque. Comparison of the angular motion of space debris was taken with and without gravitational torque taken into account and a measurement time interval was found on which the previously made assumption is applicable. The simulation results are shown in Fig. 1.

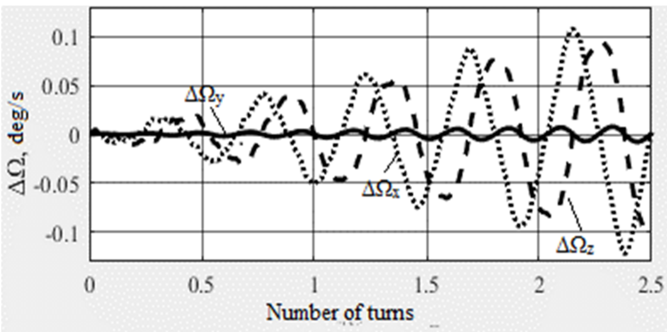

Fig. 1. Simulation results

It can be concluded from the simulation results that the difference in angular velocities $\Delta\Omega x$ can be neglected on a time interval of the order of two orbital turns, and in the measurement interval of 100 s $\Delta\Omega$ is two orders of magnitude smaller than the noise of the angular velocity sensor MPU-9250. Therefore, in the motion model, the gravitational moment is not taken into account.

The initial approximations of the orientation angles of the nanosatellite measuring axes relative to the principal axes of inertia of the orbital stage can be assumed to be zero, since according to [7] this algorithm works with an initial approximation of orientation angles ± 100%. Then, as an initial approximation for angular velocity projections to the principal axes of inertia of the orbital stage, measurements of the angular velocity projections in the axes of the nanosatellite angular velocity sensor can be used. The choice of initial approximations of the inertia coefficients requires a separate study and is discussed below.

B. *Choice of initial inertia coefficients*

In this paper, the region of initial approximations for the coefficients of inertia, in which the numerical method is convergent, is found for the second stage of the Kosmos-3M LV (for other large space debris, the region of initial approximations can be found in a similar way). The identification and study of the region of initial approximations to ensure the convergence of the algorithm are performed under the following conditions: first, the angular rotational speeds of the orbital stage are chosen randomly under the assumption of equiprobability of their distribution over the interval of values up to 72 deg/s, second, the inertia coefficients k_y, k_z move from zero to one in increments of 0.02.

Under the selected conditions, a statistical simulation of the work of the proposed approach for various combinations of the vector of initial approximations was made to isolate the domain of convergence of the method. In this case, for each combination of initial approximations, the total root-mean-square error in the approximation of the measurements of the angular velocity σ_Σ was computed. After screening out the points in which $\sigma_\Sigma < 1$ deg/s and points at which no solution was found, the applicability region of the method in coordinates (k_y, k_z) is found. The obtained region has two characteristic parts, separated by a conventionally straight line (Fig. 2).

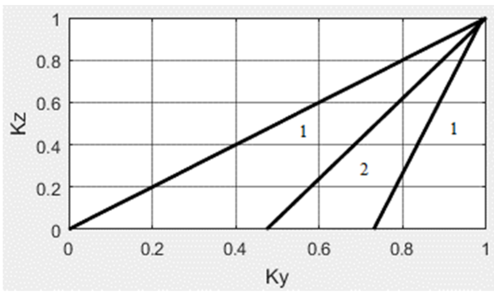

Fig. 2. Algorithm convergence area

The comparison of the number of solutions in both parts of the area shows that the probability of successful identification when selecting parameters from part 1 (Figure 2.) is 0.957, which is significantly higher than the probability 0.043 that the problem of identification cannot be solved. The right-hand side of the area also has a feature - it contains a subregion 2 (Figure 2.), in which σ_Σ <0.001 deg / s, which is obtained by screening out points that do not fit this criterion. The boundaries of a given subregion can be described by two straight lines, which are similarly determined by the method of least squares.

VI. Error Estimation of Identification of Kosmos-3M Parameters

A statistical study was made of the determination error of the inertial characteristics of the Kosmos-3M stage using on-board nanosatellite measuring instruments built on the SamSat platform under the following conditions: the noise intensity of the MPU-9250 angular velocity sensor is 0.1 deg/s [10], the initial approximations for the coefficients of inertia k_y and were chosen randomly from the selected area of convergence under the condition of equiprobable distribution law, the true values of the inertia coefficients k_y and k_z were assumed equal to the values found on the basis of the data in Table 1 (0.8 and 0.6, respectively). The number of statistical simulations was assumed to be 1000, which roughly corresponds to a 3% error in obtaining error estimates for the inertia coefficients. The distribution functions of the probabilities of errors in determining the inertia coefficients are shown in Fig. 3.

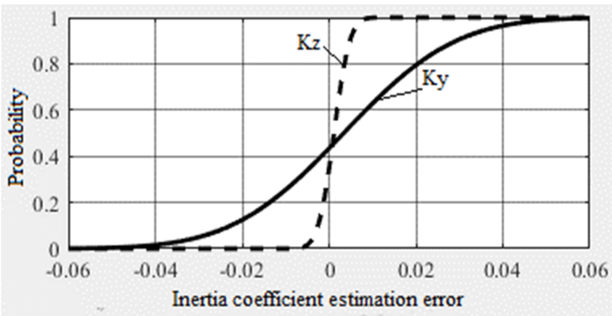

Fig. 3. Inertia coefficients error distribution

The root-mean-square deviation for k_y was 0.018, and for k_z – 0.003. The mathematical expectation of the error for k_y was 0.0023, and for k_z – 0.0011.

VII. Conclusion

Thus, the developed approach to identify inertial characteristics of bulky waste using the contact method is accurate enough and can be implemented in practice. The possibility of using nanosatellites based on the MPU-9250 platform for such space missions is confirmed by statistical modeling. Each type of bulky waste requires a special study to determine a range of initial approximations for inertia coefficients in order to find the convergence domain of the numerical Gauss-Newton method used, that does not possess the property of global convergence. If it is not possible to determine the convergence domain beforehand, consider using other numerical procedures.

References

[1] Valeriy Trushlyakov Choice of a suitable target for developing proposals for an adr flight demonstration experiment /Luciano Anselmo, Carmen Pardini // 7th European Conference on Space Debris, 2017.

[2] Lavagna, M. Uncooperative objects pose, motion and inertia tensor estimation via stereovision / V. Pesce, R. Bevilacqua // Advanced Space Technologies for Robotics and Automation, 2015.

[3] Sheinfeld, D. Rigid body inertia estimation with applications to the capture of a tumbling satellite / R. Stephen // 19th AAS/AIAA Spaceflight Mechanics Meeting, 2009.

[4] Benninghoff, H. Rendezvous involving a non-cooperative, tumbling target - estimation of moments of inertia and center of mass of an unknown target // International Symposium on Space Flight Dynamics, 2015.

[5] N.N.Sevastjyanov, V.N.Branets, Yu.R.Banit, M.Yu.Belyaev, V.V.Sazonov Севастьянов, Н.Н. Opredelenie tenzora inertsii geostatsionarnyh sputnikov Jumal" po telemetricheskoi informatsii, Preprint of KIAM RAS, 2006, no. 17, pp. 1-26.

[6] Yu.R.Banit, M.Yu.Belyaev, T.A.Dobrinskaya, N.I.Efimov,V.V.Sazonov, V.M.Stazhkov. Opredelenie tenzora inercii mezhdunarodnoj kosmicheskoj stancii po telemetricheskoj informacii, Preprint of KIAM RAS, 2006, no. 57.

[7] Lee Allan In-Flight Estimation of the Cassini Spacecraft's Inertia Tensor // Journal of Spacecraft and Rockets, 2002, no.1, pp. 153-155.

[8] Brandin, V.N. Eksperimental'naya ballistika kosmicheskih apparatov (Experimental spacecraft ballistics), / A.A. Vasil'ev, A.A. Kunitskii // Moscow: Mashinostroenie, 1984.

[9] Eddy current braking applied to the kosmos-3m second rocket stage master thesis by Michielsen

https://www.invensense.com/wp-content/uploads/2015/02/RM-MPU-9250A-00-v1.6.pdf