Estimating the Inertial Characteristics of a Nanosatellite Using a Radio Compass Based on GNSS Technology*

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***Abstract* — The paper describes a radio compass based on GNSS technology, an algorithm for processing GNSS information, an algorithm for calculating the coordinates of a unit vector of the longitudinal axis of a nanosatellite, and an approach that allows an estimation of the inertial characteristics of a nanosatellite. The estimation of the influence of the baseline length on the accuracy of determining the orientation angles is given. The results of the statistical study of the error determination of the nanosatellite inertial parameters using a radio compass are presented.**

Key words —nanosatellite, GPS, GLONASS, radio compass, navigation reciever, inertial chatacteristics, identification algorithm

I. INTRODUCTION

In this paper, an approach is proposed and investigated that allows one to estimate the inertial characteristics of a nanosatellite. The coordinates used in the approach are the coordinates of the unit vector of the nanosatellite longitudinal axis, along which the navigation receiver antennas are located, in the orbital coordinate system obtained at a certain time interval. The unit vector of the longitudinal axis can be calculated by using a radio compass based on GNSS technology. This approach can be useful in controlling the processes of opening of solar cells or engine operation, or any other operation that leads to variations of inertia moments.

II. RADIO COMPASS

A. Coordinate system

To conduct research and describe the movement of a nanosatellite, the following coordinate systems are introduced in the work (Fig. 1).

The coordinate system associated with the spacecraft (SCS) is formed by the main central inertia axis and designated as $OX_sY_sZ_s$. The absolute geocentric coordinate system (ACS) designated as $CX_aY_aZ_a$ has its origin in the Earth's center of mass (point C). Axis X_a is directed to the vernal equinox point. Axis Z_a is directed to the north pole. Axis Y_a complements the system to the right. The orbital coordinate system (OCS) is designated as $OX_oY_oZ_o$, with its origin lying in the spacecraft's center of mass. Axis X_o is directed along the nanosatellite's radius-vector. Axis Z_a is normal to the orbital plane. Axis Y_a complements the system to the right.

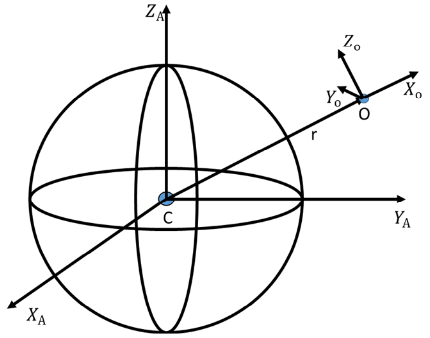

Fig. 1. Coordinate systems

Transition from ACS to OCS is set by three successive turns: first by ascending node longitude angle Ω around axis Z_a, second by orbital inclination angle i around new axis X'_a and third by the latitude argument u around new axis Z''_a. Orientation of SCS relative to OCS is set by three successive turns: first by precession angle ψ around Y_o, second by angle of attack α around new Z'_o and third by the angle of proper rotation φ around Y''_o.

B. Algorithm

The radio compass has several advantages over inertial measuring instruments: it is not sensitive to magnetic disturbances and discontinuities, like magnetic compasses; does not require constant maintenance of work, as is the case with gyroscopes; there is no accumulation of errors in the measurements, since the measurements are independent.

There are several options for building radio compasses based on GNSS technology [1,2]. The simplest option is to determine the relative position based on the calculation of the difference between the coordinates of two receivers or the difference of two consecutive coordinates, but this method has relatively low accuracy on small baselines.

Radio compass consists of two commercial navigation receivers connected to the computer. Antennas of navigation receivers are rigidly fixed at a known distance, forming a baseline. The calculator receives raw navigation measurements: pseudorange, phase measurements, navigation messages. According to the data obtained, a rough estimate of the coordinates of the base receiver is calculated. Based on the phase measurements, the relative occupancy of the second receiver relative to the first (basic) is calculated. From the data obtained, the orientation angles of the base line are formed (Fig. 2.).

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Since the phase measurements have resolution of 19 cm, ambiguities in determining distances arise. Therefore, the algorithm for calculating orientation angles consists of two stages: at the first stage, ambiguity resolution is performed by processing differential measurements of pseudoranges and minimizing the error in calculating the baseline length. At the second stage, double differences of phase measurements between the receivers and the reference navigation satellite and other navigation satellites are calculated , taking into account the ambiguities. The definition of the initial angles is performed by the least squares method.

Based on the angles obtained, the coordinates of the unit vector of the longitudinal axis of the nanosatellite are calculated. At a certain time interval, the calculated coordinates accumulate and form an array of data reflecting the orientation of the longitudinal axis in time. The dependences of the coordinates of the vector of the longitudinal axis are approximated by a measurement model. In this case, the dynamic and kinematic equations of the angular motion of the nanosatellite are used. The inertial parameters entering into dynamic equations are estimated on the basis of minimizing the square error between the measured coordinates and their model values.

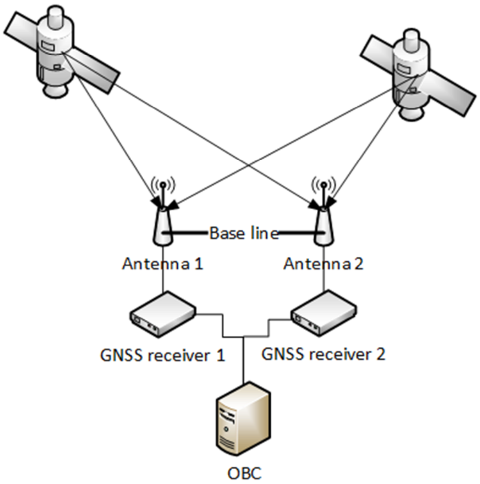

Fig. 2. Radio compass scheme

III. Parametric Research of the Accuracy of Radio Compass

Mathematical simulation was carried out to determine how the length of baseline influence on the accuracy of attitude determination. Results are shown in Fig. 3.

The research is based on measurements that were received from navigation antennas with a baseline of 0.5 m. Simulation consists of two stages. At the first stage, the relative position of the slave receiver relative to the master was calculated, based on the baseline length of 0.5 m and the coordinates of the navigation satellites. Then, pseudoranges and phase measurements were corrected on the basis of the baseline length that was changed. At the second stage, the repeated calculation of the relative position of the receivers with new initial data was made. According to the study, an orientation with an accuracy of at least 1.5 degrees can be obtained.

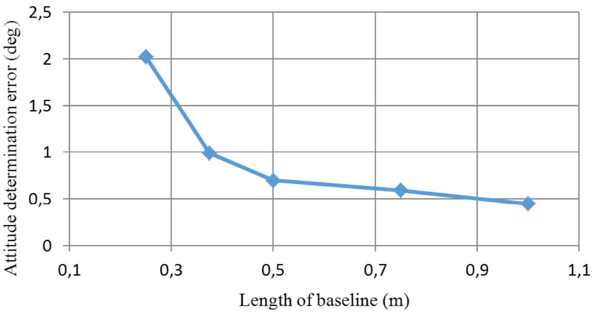

Fig. 3. Influence of baseline length on the attitude determination accuracy

IV. Estimation of Nanosatellite's Inertial Characteristics

A. Problem statement

According to the results of the radio compass algorithm simulation, there is a set of measurements of the coordinates of the unit vector of the nanosatellite's longitudinal axis in the OCS. Measurements were obtained at a certain time interval T. It is required to obtain estimates of the nanosatellite's inertial moments at a given time interval. This task is reduced to the problem of nonlinear multi-parameter optimization, namely, to search for the minimum of the following objective function:

$$J(b) = \sum_{\alpha=xyz} \sum_{i=1}^{N} \left(\zeta_{\alpha i}(b) - \eta_{\alpha i}\right)^2 \qquad (1)$$

where $\zeta_{\alpha i}$ - measurement model of the unit vector of the longitudinal axis of the nanosatellite in the OCS, b - estimated parameters vector, $\eta_{\alpha i}$ – calculated coordinates of the unit vector of the longitudinal axis. Estimated parameters vector has a form:

$$b = \left[\omega_x(t_0), \omega_y(t_0), \omega_z(t_0), \psi(t_0), \alpha(t_0), \varphi(t_0), \lambda, \mu, C_{ma}\right] \qquad (2)$$

where $\psi(t_0)$ – precession angle, $\alpha(t_0)$ – angle of attack, $\varphi(t_0)$ – angle of proper rotation, $\omega_x(t_0)$, $\omega_y(t_0)$, $\omega_z(t_0)$ – angular velocities, t_0 – starting time. Unlike [3], if estimates of λ, μ, C_{ma} are known it is possible to возможно calculate the moments of inertia explicitly.

The problem should be solved under the following assumptions

- It is assumed that the nanosatellite is equipped with satellite navigation antennas that have a hemisphere-free beam pattern, which excludes the possibility of receiving a signal from navigation satellites located outside the hemisphere.
- The module of the angular velocity of the nanosatellite does not exceed 3 deg/s, which is selected from the condition of guaranteed receipt of the full frame of navigation information.
- The center of mass of the nanosatellite is shifted only along the longitudinal axis.

B. Motion model

The rotational motion of a nanosatellite is described by the dynamic Euler equations (3) and kinematic equations (4) [3]. In the right parts of the dynamic Euler equations, the gravitational and aerodynamic moments are have a following form:

$$\begin{aligned}
\dot{\omega}_x &= \mu\left(\omega_y\omega_z - \nu\, a_{21}a_{31}\right) + \left(r_y V_{cz} - r_z V_{cy}\right) S\left|V_c\right| C_{ma}\rho \\
\dot{\omega}_y &= \frac{1-\lambda}{1+\lambda\mu}\left(\omega_x\omega_z - \nu\, a_{31}a_{11}\right) + \frac{\lambda}{1+\lambda\mu}\left(r_z V_{cx} - r_x V_{cz}\right) S\left|V_c\right| C_{ma}\rho \\
\dot{\omega}_z &= -\left(1-\lambda+\lambda\mu\right)\left(\omega_x\omega_y - \nu\, a_{21}a_{11}\right) + \lambda\left(r_x V_{cy} - r_y V_{cx}\right) S\left|V_c\right| C_{ma}\rho
\end{aligned} \tag{3}$$

where $\lambda = I_x/I_z$ and $\mu = (I_y - I_z)/I_x$ – dimensionless (expressions for dimensionless inertia coefficients are taken from [3]); $\nu = 3\cdot\mu_e/r^3$ – gravitational moment coefficient; $C_{ma} = C_x/(2I_x)$ – aerodynamic moment coefficient; S – the area of the projection of the nanosatellite surface onto a plane perpendicular to the velocity vector; ρ – atmospheric density at the altitude of the nanosatellite; $\overrightarrow{V_c} = \left(V_{cx}\, V_{cy}\, V_{cz}\right)$ – orbital velocity vector in SCS; $\vec{r} = \left(r_x\, r_y\, r_z\right)$ – vector directed from center of mass to center of pressure in SCS.

$$\begin{aligned}
\dot{q}_0 &= 0.5\cdot\left(-\left(\omega_x - \omega_{rx}\right)\cdot q_1 - \left(\omega_y - \omega_{ry}\right)\cdot q_2 - \left(\omega_z - \omega_{rz}\right)\cdot q_3\right) \\
\dot{q}_1 &= 0.5\cdot\left(\left(\omega_x - \omega_{rx}\right)\cdot q_0 + \left(\omega_z - \omega_{rz}\right)\cdot q_2 - \left(\omega_y - \omega_{ry}\right)\cdot q_3\right) \\
\dot{q}_2 &= 0.5\cdot\left(\left(\omega_y - \omega_{ry}\right)\cdot q_0 + \left(\omega_x - \omega_{rx}\right)\cdot q_3 - \left(\omega_z - \omega_{rz}\right)\cdot q_1\right) \\
\dot{q}_3 &= 0.5\cdot\left(\left(\omega_z - \omega_{rz}\right)\cdot q_0 + \left(\omega_y - \omega_{ry}\right)\cdot q_1 - \left(\omega_x - \omega_{rx}\right)\cdot q_2\right)
\end{aligned} \tag{4}$$

where $\vec{\omega}_r = A(q)\left[0 \quad 0 \quad \omega_{\text{орб}}\right]^T$, ω_{orb} – angular orbital velocity. $A(q)$ – transition matrix from OCS в SCS.

C. Statistical study of the accuracy of the solution for identifying the inertial characteristics of a nanosatellite

To determine the accuracy of the estimation of the moments of inertia of a nanosatellite, a statistical study of the solution of the problem was carried out. 400 variants of the initial conditions of angular motion were randomly generated $\left(\omega_x(t_0), \omega_y(t_0), \omega_z(t_0), \psi(t_0), \alpha(t_0), \varphi(t_0)\right)$ under the assumption of a uniform law of their distribution. This amount allows to get statistical characteristics with an accuracy of 5%. It was set that $\left|\vec{\omega}\right| \le 3\, deg/s$, $\psi(t_0)$ and $\varphi(t_0)$ are in the range from 0 to 360°, $\alpha(t_0)$ - from 0 to 180°. Measurements of the radio compass were simulated for each variant of the initial conditions,. The measurements were used to reconstruct the moments of inertia of the nanosatellite. The results of solving the problem were compared with the initial conditions that were used to simulate the measurements. As a result, the relative errors of the algorithm for estimating the moments of inertia of a nanosatellite were obtained.

The study is given for altitudes of 300, 450 and 600 km, since, according to [4], the ratio of aerodynamic and gravitational moments changes at these altitudes. The simulation results are shown in Fig.4, Fig.5 and Fig.6.

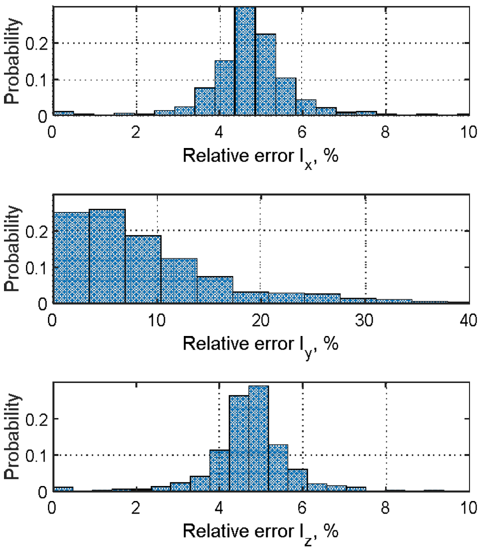

Fig. 4. Results for altitude 300 km

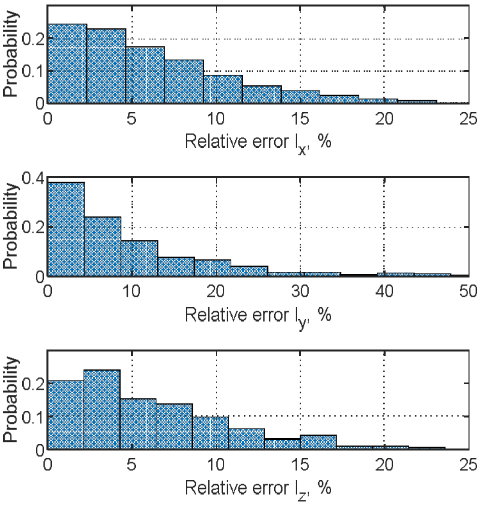

Fig. 5. Results for altitude 450 km

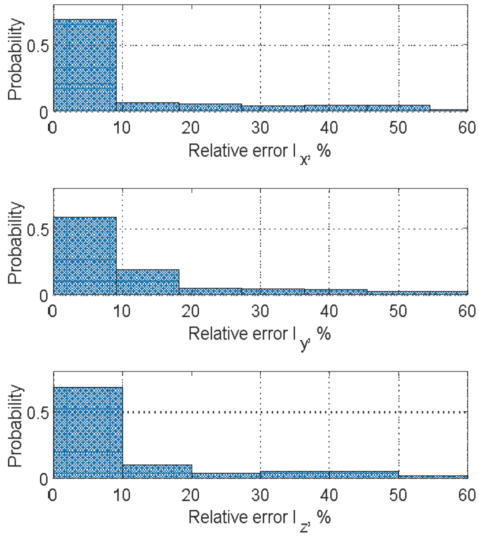

Fig. 6. Results for altitude 600 km

According to the obtained results, the following conclusions can be made: identification of the moments of inertia in an explicit form is possible only with a significant prevalence of the aerodynamic moment; the error in estimating the longitudinal moment of inertia is two times higher than the errors in estimating the transverse moments of inertia this is due to the fact that the aerodynamic moment does not act along the longitudinal axis of the nanosatellite.

V. CONCLUSION

The possibility of using a radio compass in the problem of identifying the inertial characteristics of a nanosatellite is shown. The accuracy of determining orientation using a radio compass was experimentally shown.

REFERENCES

[1] Borre, K., Strang G., Algorithms for global positioning: Wellesley-Cambridge press, 2012.-433 p.

[2] http://www.javadgnss.ru/about/news/justin-algoritmi-i-priemi

[3] Abrashkin V.I., Voronov K.E., Piyakov A.V., Puzin YU.YA., Sazonov V.V., Semkin N.D., Filippov A.S., CHebukov S.YU., Determination of the spacecraft Aist attitude motion on measurements of the Earth magnetic field, Preprint of KIAM RAS, 2014, no. 17.

Belokonov I. V., Timbai I. A., Nikolaev P. N., Analysis and Synthesis of Motion of Aerodynamically Stabilized Nanosatellites of the CubeSat Design, Gyroscopy and Navigation, 2018, Vol. 9, No. 4, pp. 287–300.