Passive Stabilization Systems for CubeSat Nanosatellites: General Principles and Features

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***Abstract*—This paper studies the dynamics of the angular motion of CubeSat nanosatellites with the passive stabilization systems of different types: aerodynamic, aerodynamic-gravitational, gravitational, gravitational-aerodynamic systems. The research was carried out in a probabilistic formulation. Using the obtained analytical distribution functions of the maximum angle of the nanosatellite longitudinal axis deviation from the required direction (orbital velocity vector, or local vertical), the formulas for selecting design parameters (geometrical dimensions, static stability margin, inertia moments) are derived. In the circular orbit at the required flight altitude, these parameters provide the deviation of the longitudinal axis from the required direction less than the allowable value with a given probability for a specified error of the initial angular velocity formed by the separation system. The nomograms for selecting the main design parameters of a CubeSat nanosatellite have been constructed.**

Keywords*—*CubeSat nanosatellite, aerodynamic moment, gravitational moment, angle of attack, passive stabilization systems

I. Introduction

Most of the space researches conducted with the use of nanosatellites require the knowledge of their specific orientation in space. To ensure a given orientation of nanosatellites in space, passive or combined (passive in combination with active) stabilization systems are generally used. They do not require or require small expenditure of the working body and the energy stored on board. One of the most important tasks is studying the uncontrolled motion of a nanosatellite with respect to the center-of-mass due to the fact that the design conditions of attitude motion of a nanosatellite with passive stabilization system can be ensured only at the design stage by choosing its design parameters, as well as specifying limitation on the angular velocity generated by the separation system, and for combined stabilization system at the end of the operation of the preliminary damping system.

When developing a passive stabilization system, as a rule, knowledge is used about stable equilibrium positions of a nanosatellite due to the action of external moments. Most nanosatellites are launched into the low circular orbits, where gravitational and aerodynamic moments dominate and it is advisable to use both of the moments to stabilize the angular position.

In the known publications, the problem of ensuring required stabilization of nanosatellites is solved in a deterministic formulation. For example, in [1] the problem of aerodynamic stabilization of a CubeSat was solved by deploying solar panels at a certain angle to the longitudinal axis after separation from the launcher.

In this paper, based on the analysis of the nanosatellite motion in low orbits, the authors consider the problem of stabilization in a probabilistic formulation with respect to the angular motion of the nanosatellite after separation from the launcher. This paper summarizes and supplements previous studies of methods for passive stabilization of CubeSat nanosatellites [25]. It expands the use of passive stabilization systems for the two most popular orientations of the nanosatellite (vertical and vector orbital motion) to altitudes up to 700 km. It is the maximum altitude of the guaranteed descent from orbit in a time not exceeding 25 years, in the absence of special removal facilities. This study has allowed to introduce a classification of passive stabilization systems for CubeSat in a circular orbit: aerodynamic, aerodynamic-gravitational, gravitational, and gravitational-aerodynamic which can be chosen in relation to the altitude range of the dominance of a certain type of external moments and a type of stabilization (one-axis and three-axis).

This classification of stabilization systems for CubeSat is based on the fact that the angular acceleration of a nanosatellite caused by the aerodynamic moment is two order higher than that for a satellite with large dimensions and mass (with the same values of the relative static stability margin and mass density value) [5]. This extends the range of altitudes at which the aerodynamic moment acting on the nanosatellite is significant and together with the gravitational moment it can be used for passive stabilization of the angular motion.

Fig. 1 shows the areas of altitudes H and the relative static stability margins $\Delta\bar{x} = \Delta x / l$ (Δx is the distance from the center-of-mass to the geometric center of the nanosatellite, l is the characteristic length of the nanosatellite) for the different types of the passive stabilization: area 1 is the area of the one-axis aerodynamic stabilization along the velocity vector (it is the area where the aerodynamic moment dominates over the gravitational one $M_a \geq 2M_g$); area 2 is area of the three-axis aerodynamic-gravitational stabilization (it is the area where the aerodynamic moment dominates over the gravitational one $M_a \geq 2M_g$); area 3 is area of the one-axis gravitational stabilization along the local vertical (it is the area where the gravitational moment dominates over the aerodynamic one $M_g \geq 5M_a$); area 4 is the area of the three-axis gravitational-aerodynamic stabilization (it is the area of the any magnitudes of aerodynamic and gravitational moments). For calculations the boundaries of areas it was used the standard model of the atmospheric density [6]. It should be noted, that the size of the areas may vary in depending on the level of solar activity.

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II. Aerodynamic Stabilization along the Velocity Vector

It is considered the choice of design parameters for a dynamically symmetric CubeSat nanosatellite to provide the passive stabilization the longitudinal axis along the orbital velocity vector (area 1 in Fig. 1).

When the nanosatellite is separated from the deployer, the value of the maximum angle of attack has a random nature. The latter is determined not only by the values of the aerodynamic and gravitational moments but the initial values of the angle of attack (α_0) and angular velocity ($\dot{\alpha}_0$) too. Assuming that the magnitude of the initial angular velocity ($\dot{\alpha}_0$) has the greatest spread, and neglecting the spreads other parameters, it was obtained in [7] the distribution functions of the maximum angle of attack ($\alpha_{\max}$) at the time of separation from the deployer.

If the modulus of $\dot{\alpha}_0$ is distributed according to the Rayleigh law, then the distribution function of the maximum angle of attack is determined by the formula [9]:

$$F(\alpha_{\max}) = 1 - e^{\frac{-a(\cos\alpha_{\max} - \cos\alpha_0) - c(\cos^2\alpha_{\max} - \cos^2\alpha_0)}{\sigma^2}} \quad (1)$$

where $\sigma > 0$ is the scale distribution parameter, $a(H) = a_0 Slq(H)/J$ is the coefficient due to the aerodynamic restoring moment (for CubeSat it is $a_0 = -c_0 \Delta\bar{x} 4k/\pi$; $c_0 = 2.2$ is the drag coefficient; k is the ratio of the one lateral side surface area to the characteristic area); S is the characteristic area; J is the nanosatellite transverse moment of inertia; J_x is the nanosatellite's longitudinal moment of inertia; $q(H) = V^2\rho(H)/2$ is the velocity head; V is the satellite flight velocity; H is the flight altitude, $\rho(H)$ is the atmosphere density; $c(H) = 3(J - J_x)(\omega(H))^2/(2J)$ is the coefficient due to the gravitational moment; $\omega(H) = \sqrt{\mu/(R_E + H)^3}$ is the nanosatellite center-of-mass angular velocity in orbit; R_E is the Earth's radius; μ is the Earth's gravity parameter.

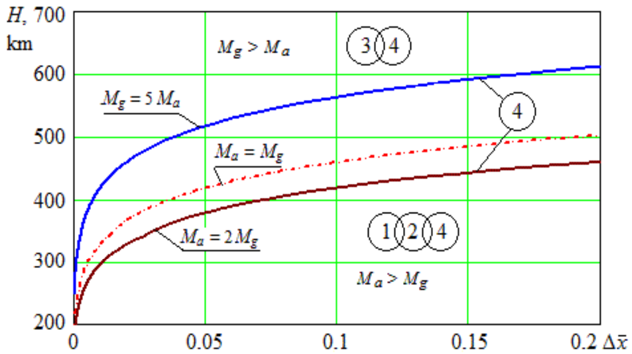

Fig. 1. The regions of a preferable passive stabilization of 3U CubeSat depending on the altitude H and relative stability margin $\Delta\bar{x}$

If the modulus of $\dot{\alpha}_0$ is distributed uniformly in the range of $[0, \dot{\alpha}_{0\max}]$, then the cumulative distribution function of the maximum angle of attack is determined by the formula [9]:

$$F(\alpha_{\max}) = \frac{\sqrt{2a(\cos\alpha_{\max} - \cos\alpha_0) + 2c(\cos^2\alpha_{\max} - \cos^2\alpha_0)}}{\dot{\alpha}_{0\max}} \quad (2)$$

Using coefficient a due to the aerodynamic moment in (1) and (2) and neglecting coefficient c due to the gravitational moment, it is derived the formulas for the constructive parameter d. To fulfill the condition the maximum angle of attack of a CubeSat $\alpha_{\max}$ to be less than the permissible value with a probability of no less than p^*, the following requirements for the constructive parameter d have to be fulfilled [5]:

- in the case when the value of the initial angular velocity corresponds to the Rayleigh distribution

$$d = \frac{\Delta x}{J} lb \geq \frac{\pi\sigma^2 \ln(1 - p^*)}{4c_0\left(\cos\alpha^*_{\max} - \cos\alpha_0\right)q(H)}; \quad (3)$$

- in the case when the initial angular velocity is distributed in accordance with the uniform law within the range $[0, \dot{\alpha}_{0\max}]$

$$d = \frac{\Delta x}{J} lb \geq \frac{\pi\left(\dot{\alpha}_{0\max} p^*\right)^2}{8c_0\left(\cos\alpha_0 - \cos\alpha^*_{\max}\right)q(H)}, \quad (4)$$

where b is the side of the rectangular parallelepiped base.

Using (3) and (4), it is possible to construct nomograms to estimate the possibility of providing the required value of the constructive parameter d. For example, the right-hand part in Fig.2 shows the dependencies of the required constructive parameter d of the nanosatellite on the orbit altitude H and the value of parameter σ (the value of the initial transverse angular velocity has the Rayleigh distribution) for the values of the maximum angle of attack $\alpha^*_{\max} = 20$ deg, the probability $p^* = 0.95$ and the initial angle of attack $\alpha_0 = 0$. The left-hand part in Fig. 2 gives the dependences of the 3U CubeSat constructive parameter values for different magnitudes of the transverse moment of inertia on the static stability margin Δx.

Nomograms can be used both to select the design parameters of the nanosatellite and to specify the requirements for the variation of the initial longitudinal angular velocity. In particular, Fig. 2 shows a sequence of choosing the nanosatellite parameters for the orbit altitude H = 380 km with the given constraints on the attitude motion formed by the target flight task $\alpha^*_{\max} = 20$ deg, $p^* = 0.95$, $\alpha_0 = 0$, $\sigma = 0.05$ deg/s. As can be seen, the value of the nanosatellite constructive parameter needed to ensure the specified motion must meet the condition of $d \geq 0.13$ m/kg (right side of the figure). The design parameters are selected on the basis of the left part of the figure.

If the aerodynamic moment is slightly larger than the gravitational one, then using formulas (1) and (2), it is possible to reassess the probability of meeting the requirements for the maximum angle of attack or to set new limits on the angular velocities generated by the separation system, or, in the case of the combined stabilization system, at the end of the operation of the preliminary damping system.

In the case when the value of the initial angular velocity ($\dot{\alpha}_0$) corresponds to the Rayleigh distribution, the restriction on the scale distribution parameter has the form:

$$\sigma \le \sqrt{\frac{-a(\cos\alpha^*_{\max} - \cos\alpha_0) - c(\cos^2\alpha^*_{\max} - \cos^2\alpha_0)}{\ln(1-p^*)}} \qquad (5)$$

In the case when the initial angular velocity ($\dot{\alpha}_0$) is distributed uniformly within the range $[0, \dot{\alpha}_{0\max}]$, the restriction on $\dot{\alpha}_{0\max}$ is determined by the formula:

$$\dot{\alpha}_{0\max} \le \frac{\sqrt{2a(\cos\alpha^*_{\max} - \cos\alpha_0) + 2c(\cos^2\alpha^*_{\max} - \cos^2\alpha_0)}}{p^*} \qquad (6)$$

The proposed approach of choosing design parameters of the aerodynamically stabilized CubeSat nanosatellite was granted the Eurasian patent [8]. This approach was used for the design of two nanosatellites in Samara National Research University. The first nanosatellite SamSat-218D [9] was designed to develop and test the technology of creating a closed control loop for its spatial orientation with a large static stability margin. The second nanosatellite SamSat-QB50 [10] was designed as part of an international university project QB50 and intended to study the Earth's thermosphere as part of nanosatellite constellation. The technology for synthesizing design parameters used in this project was based on the artificial creation of the required static stability margin due to the transformation of the structure and deployment of an aerodynamic stabilizer.

III. Three-Axis Aerodynamic-Gravitational Stabilization

Fig. 1 shows the area of possible implementation of a three-axis aerodynamic-gravitational stabilization of a nanosatellite (area 2) in low circular orbits for the case when the aerodynamic moment determines the attitude motion of a nanosatellite. This moment ensures the stabilization of the nanosatellite longitudinal axis. The stabilization of the transverse axes of the nanosatellite is carried out due to the gravitational moment.

Stabilization of the longitudinal axis of the nanosatellite along velocity vector can be provided by the selection of its design parameters (statical stability margin, geometric dimensions, and greatest moment of inertia J_y) as shown above. At the same time, the possibility of stabilizing the transverse axes of a nanosatellite can be achieved by the gravitational moment [4].

Let us assume that the value of the initial longitudinal angular velocity (ω_{x0}) corresponds to a normal distribution with mean zero and standard deviation σ. Then the cumulative distribution function of the value of the maximum roll angle $\delta_{\max}$ (it is the angle of deviation of the transverse axis Oz, for which the value of the moment of inertia J_z takes a value that satisfies the condition $J_x < J_z < J_y$) has the form:

$$F(\delta_{\max}) = 2\Phi_0\left(\frac{\sqrt{2d_0(\cos 2\delta_{\max} - \cos 2\delta_0)}}{\sigma}\right) \qquad (7)$$

where $\Phi_0(t) = \frac{1}{\sqrt{2\pi}}\int_0^t e^{-t^2/2}dt$ is the normal distribution function, $d_0 = \frac{3\mu}{2(R_3+H)^3}\left(\frac{J_z - J_y}{J_x}\right)$.

Let us assume that the modulus of ω_{x0} has a distribution according to the uniform law in the range $[0, \omega_{x0\max}]$, then the cumulative distribution function of the modulus of $\delta_{\max}$ is given by:

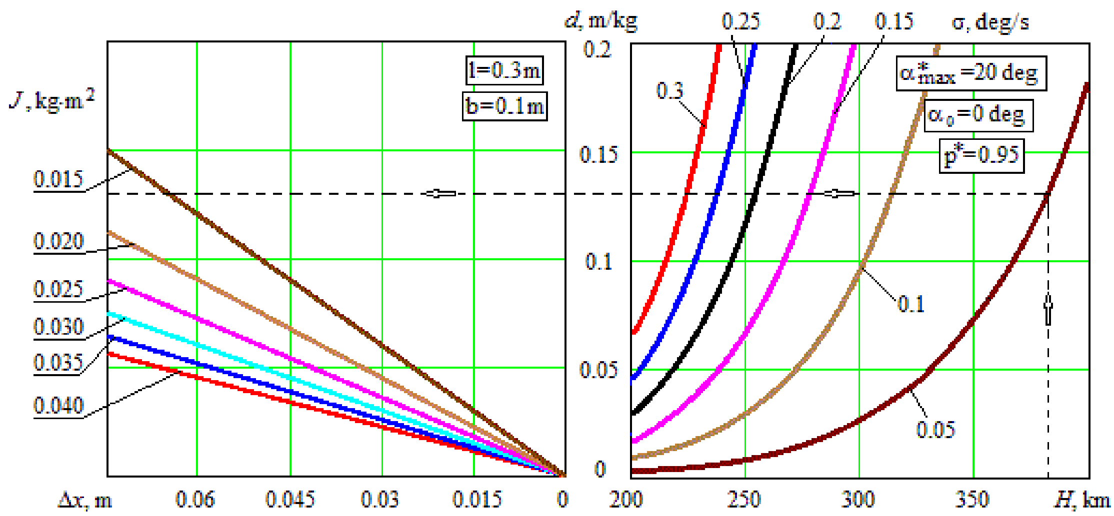

Fig. 2. The nomogram for choosing the constructive parameter of the nanosatellite with the aerodynamic stabilization system

$$F(\delta_{max}) = \frac{\sqrt{2d_0(\cos 2\delta_{max} - \cos 2\delta_0)}}{\omega_{x0}}. \quad (8)$$

Setting p^* as the probability of realizing the allowable value of the maximum roll angle δ_{max} and solving expressions (7), (8) with respect to the design parameters combined in the constructive parameter $d_k = \frac{J_y - J_z}{J_x}$ of the nanosatellite, is obtained a requirement for its magnitude. In order to reach the condition that the maximum roll angle δ_{max} should be less than the permissible value for the given spread in the longitudinal angular velocity generated by the separation system (or, in the case of the combined stabilization system, at the end of the operation of the preliminary damping system) with a probability of no less than p^*, it is necessary to fulfill the following condition for the constructive parameter of the nanosatellite d_k:

- if the value ω_{x0} has the normal distribution with mean zero and standard deviation σ:

$$d_k = \frac{J_y - J_z}{J_x} \geq \frac{(R_3 + H)^3}{3\mu} \frac{\sigma^2 (t^*)^2}{(\cos 2\delta_0 - \cos 2\delta_{max})}, \quad (9)$$

where t^* is the argument of the normal distribution function for the given probability: $\Phi_0(t^*) = p^*/2$;

- in the case of the distribution of the modulus of the initial longitudinal angular velocity ω_{x0} according to the uniform law in the range $[0, \omega_{x0\max}]$:

$$d_k = \frac{J_y - J_z}{J_x} \geq \frac{(R_3 + H)^3}{3\mu} \frac{(\omega_{x0\max} p^*)^2}{(\cos 2\delta_0 - \cos 2\delta_{max})}. \quad (10)$$

Using the obtained expressions (9), (10), it is possible to construct nomograms for estimating the possibility of providing the required value of the constructive parameter d_k.

For example, the right-hand part in Fig. 3 shows the dependencies of the required constructive parameter of the nanosatellite d_k on the permissible roll angle δ^*_{max} and on the value of the initial longitudinal angular velocity $\omega_{x0\max}$ (the initial longitudinal angular velocity is distributed uniformly). The nomograms were calculated for the initial roll angle $\delta_0 = 0$, the probability $p^* = 0.95$, and altitude of the flight $H = 380$ км. The left-hand part in Fig. 3 shows the values of the constructive parameter of the nanosatellite d_k depending on the values of the moments of inertia J_z and J_x for the greatest moment of inertia $J_y = 0.025\ \text{kg}\cdot\text{m}^2$.

The nomograms can be used both to select the design parameters of the nanosatellite and to specify the requirements for the deviation of the initial longitudinal angular velocity.

IV. Gravitational Stabilization along the Local Vertical

Let us consider the choice of design parameters for a dynamically symmetric CubeSat nanosatellite to provide a gravitational passive stabilization system for its longitudinal axis along the local vertical (area 3 in Fig. 1).

This type of passive one-axis stabilization of a dynamically symmetric nanosatellite is applicable for a range of orbits in which the gravitational moment dominates and seeks to orient the nanosatellite in a way that the axis of the smallest moment of inertia coincides with the local vertical.

Using coefficient c due to the gravitational moment in (1) and (2) and neglecting coefficient a due to the aerodynamic moment, we derived the formulas for selecting design parameters (moments of inertia) which were combined in one constructive parameter $d_g = \frac{J_x}{J}$. In order for the maximum angle of the longitudinal axis deviation from the gravitational vertical β_{max} ($\beta_{max} = \alpha_{max} - \pi/2$) to be less than the permissible value β^*_{max} with a probability of no less than p^*, the following condition for the constructive parameter of the nanosatellite has to be fulfilled [5]:

- in the case when the value of the initial angular velocity corresponds to the Rayleigh distribution:

$$d_g = \frac{J_x}{J} \leq 1 - \frac{4(R_3 + H)^3}{3\mu} \frac{\sigma^2 \ln(1 - p^*)}{(\cos 2\beta^*_{max} - \cos 2\beta_0)}; \quad (11)$$

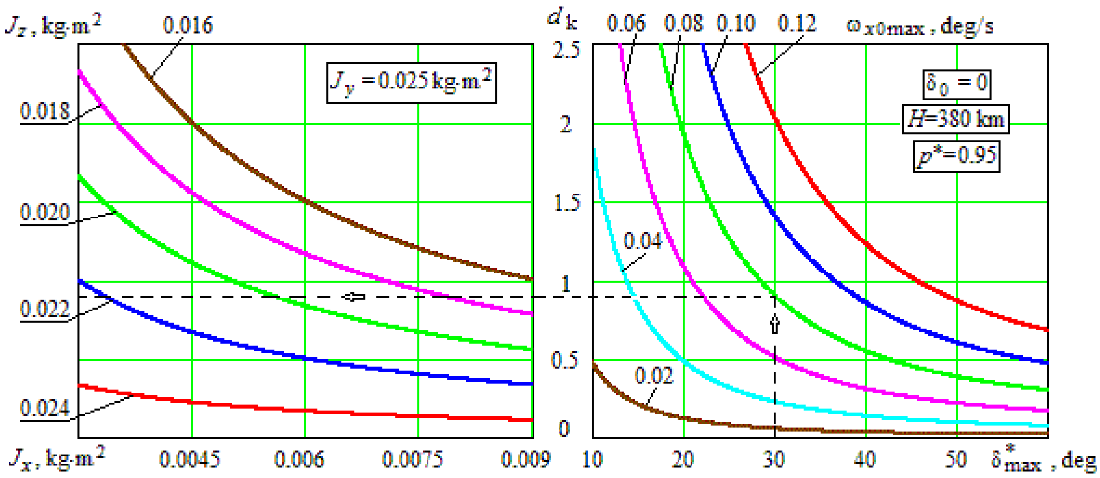

Fig. 3. The nomogram for choosing the constructive parameter of the nanosatellite with the aerodynamic-gravitational stabilization system

- in the case when the initial angular velocity is distributed in accordance with the uniform law within the range $[0, \dot{\beta}_{0\max}]$:

$$d_g = \frac{J_x}{J} \le 1 - \frac{4(R_3+H)^3}{3\mu}\frac{(\dot{\beta}_{0\max}p^*)^2}{-2\left(\cos 2\beta^*_{\max} - \cos 2\beta_0\right)}, \quad (12)$$

where β_0 is initial value of the longitudinal axis deviation from the gravitational vertical.

Formulas (11) and (12) allow constructing nomograms to assess the possibility of providing the required value of the constructive parameter d_g. For example, the right-hand part in Fig. 4 shows the dependence of required constructive parameter on the maximum permissible value of the deviation of the longitudinal axis from the local vertical $\beta^*_{\max}$ and parameter σ (the initial angular velocity corresponds to the Rayleigh distribution), the probability $p^* = 0.95$, $\beta_0 = 2\deg$, $H_0 = 350$ km. The left -hand part in Fig. 4 shows the values of the constructive parameter of the gravitationally stabilized CubeSat nanosatellite with the different values of the longitudinal and transversal moments of inertia.

In case the gravitational moment is slightly larger than the aerodynamic one, it is possible to reassess the probability of fulfilling the requirements for the maximum deviation angle of the longitudinal axis of the nanosatellite from the gravitational vertical using formulas (1) and (2), assuming $\alpha_{\max} = \pi/2 + \beta_{\max}$ or to set new limits on the angular velocities generated by the separation system (or in case of combined system of stabilization, at the end of operation of the active preliminary damping system).

In the case, when the value of the initial angular velocity $\dot{\beta}_0$ corresponds to the Rayleigh distribution, the restriction on the scale distribution parameter σ has the form

$$\sigma \le \sqrt{\frac{a(\sin\beta^*_{\max} - \sin\beta_0) - c(\sin^2\beta^*_{\max} - \sin^2\beta_0)}{\ln(1-p^*)}} \quad (13)$$

In the case, when the value of the initial angular velocity $\dot{\beta}_0$ is distributed uniformly in the range $[0, \dot{\beta}_{0\max}]$, the restriction on the value $\dot{\beta}_{0\max}$ has the form:

$$\dot{\beta}_{0\max} \le \frac{\sqrt{-2a(\sin\beta^*_{\max} - \sin\beta_0) + 2c(\sin^2\beta^*_{\max} - \sin^2\beta_0)}}{p^*} \quad (14)$$

V. Three-Axis Gravitational-Aerodynamic Stabilization

Three-axis gravitational-aerodynamic stabilization is shown as area 4 in Fig.1. It is known that the gravitational moment tends to orient nanosatellite, so that the axis of the least moment of inertia (longitudinal axis) coincides with the local vertical, the axis of the greatest moment of inertia coincides with perpendicular to the orbit plane and the axis of the intermediate moment of inertia coincides with the direction of motion.

The novelty of the work is that the passive three-axis gravitational aerodynamic stabilization is carried out by certain shifting the center-of-mass from the center of pressure on the principal axis with the intermediate value of the nanosatellite inertia moment. Herewith the design parameters (static stability margin, moments of inertia) of gravitationally-aerodynamically stabilized nanosatellite are chosen in a such way as to provide the deviation value of the longitudinal axis of the nanosatellite from the local vertical less than acceptable with a given probability at a known attitude and initial transverse angular velocity errors from separation system, (or in case of combined system of stabilization, at the end of operation of the active preliminary damping system) [2].

Earlier in [2] the authors obtained the law of distribution of the maximum angle of attack $\alpha_{\max} = \pi/2 + \beta_{\max}$, where $\beta_{\max}$ is the maximum angle of the longitudinal axis deviation from the gravitational vertical.

If the modulus of $\dot{\alpha}_0$ is distributed according to the

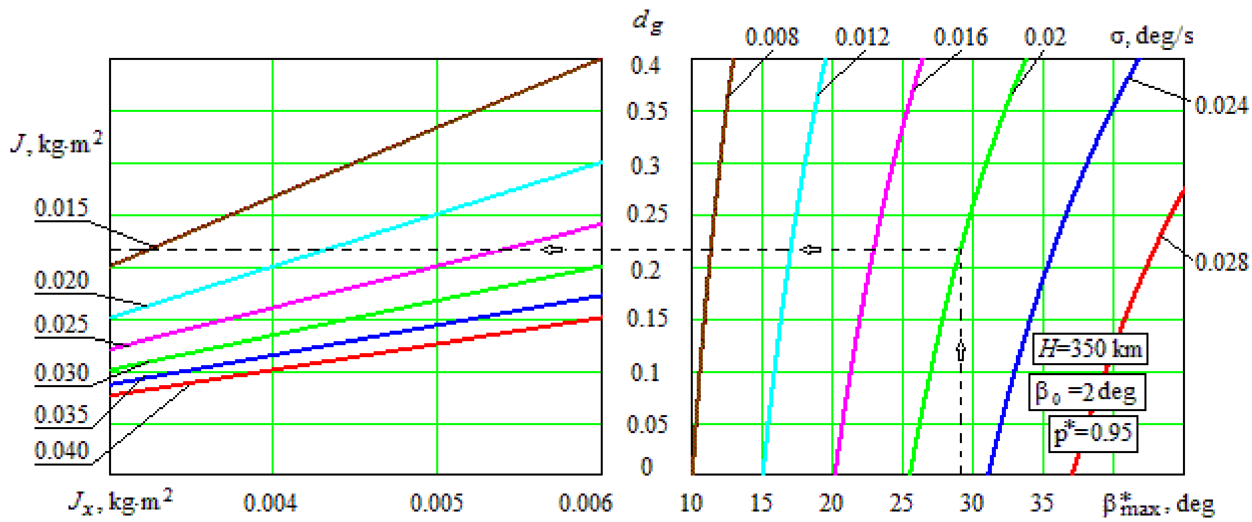

Fig. 4. The nomogram for choosing the constructive parameter of the nanosatellite with the gravitational stabilization system

Rayleigh law, then the distribution function of the maximum angle of attack is determined by the formula:

$$F(\alpha_{max}) = 1 - e^{\frac{a_z(u(\alpha_{max})-u(\alpha_0)) - a_x(v(\alpha_{max})-v(\alpha_0))}{\sigma^2}} \times e^{\frac{-c(\cos^2\alpha_{max}-\cos^2\alpha_0)}{\sigma^2}}, \quad (15)$$

where $a_z(h) = \Delta\bar{z}c_0 Slq(h)/J_y$ is the coefficient due to the aerodynamic restoring moment, because the center-of-mass is displaced along the axis with intermediate value of the moment of inertia J_z; $\Delta\bar{z} = \Delta z / l$ is the relative static stability margin along the Oz-axis; $a_x(h) = -\Delta\bar{x}c_0 Slq(h)/J_y$ is the coefficient due to the aerodynamic restoring moment, because the center-of-mass is displaced along the longitudinal axis;

$$u(\alpha) = \frac{1}{2}sign(\cos(\alpha))\cos^2\alpha + \frac{k}{2}sign(\sin(\alpha))\left(\frac{\sin 2\alpha}{2} - \alpha + 2\pi\cdot\left\lfloor\frac{\alpha+\pi}{2\pi}\right\rfloor\right),$$

$$v(\alpha) = \frac{1}{2}sign(\cos(\alpha))\left(\frac{\sin 2\alpha}{2} + \alpha - \frac{\pi}{2} - 2\pi\cdot\left\lfloor\frac{\alpha+\pi/2}{2\pi}\right\rfloor\right) + \frac{k}{2}sign(\sin(\alpha))\sin^2\alpha,$$

$\lfloor x \rfloor$ is equal to $floor(x)$,

If the modulus of $\dot{\alpha}_0$ is distributed uniformly in range $[0, \dot{\alpha}_{0\max}]$ the distribution function of the maximum angle of attack is determined by the formula:

$$F(\alpha_{max}) = [2a_z(u(\alpha_{max}) - u(\alpha_0)) - 2a_x(v(\alpha_{max}) - v(\alpha_0)) - 2c(\cos^2\alpha_{max} - \cos^2\alpha_0)]^{1/2} / \dot{\alpha}_{0\max}. \quad (16)$$

Setting p^* as the probability of realizing the allowable value of the maximum angle of attack α^*_{max}, and solving expressions (15), (16) with respect to the design parameters combined in the constructive parameter $d_1 = \frac{\Delta z}{J_y}$, we obtained a requirement for its magnitude:

- in the case when the value of the initial angular velocity $\dot{\alpha}_0$ corresponds to the Rayleigh distribution:

$$d_1 = \frac{\Delta z}{J_y} \geq [-\ln(1-p^*)\sigma^2 - c(\cos^2\alpha^*_{max} - \cos^2\alpha_0) + a_x(v(\alpha^*_{max}) - v(\alpha_0))]/[c_0 S_0 q(u(\alpha^*_{max}) - u(\alpha_0))]; \quad (17)$$

- in the case when the initial angular velocity is distributed uniformly within the range $[0, \dot{\alpha}_{0\max}]$:

$$d_1 = \frac{\Delta z}{J_y} \geq [(\dot{\alpha}_{0\max} p^*)^2 - 2c(\cos^2\alpha^*_{max} - \cos^2\alpha_0) + 2a_x(v(\alpha^*_{max}) - v(\alpha_0))]/[c_0 S_0 q(u(\alpha^*_{max}) - u(\alpha_0))]; \quad (18)$$

Formulas (17) and (18) allow constructing nomograms to assess the possibility of providing the required value of the constructive parameter d_1. For example, the right-hand part in Fig. 5 shows the dependence of required constructive parameter on the orbit altitude H and on the parameter σ (the initial angular velocity corresponds to the Rayleigh distribution) for the values of the maximum deviation of the longitudinal axis from the local vertical $\beta^*_{max} = 30$ deg ($\alpha^*_{max} = 120$ deg), the probability $p^* = 0.95$ and an initial angle of attack $\alpha_0 = 95$ deg ($\beta_0 = 5$ deg), considering the relative static stability margin by the axis Ox $\Delta\bar{x} = 0.0033$ and values of the moments of inertia are equal: $J_x = 0.006\,\text{kg}\cdot\text{m}^2$, $J_z = 0.018\,\text{kg}\cdot\text{m}^2$. The left-hand part in Fig. 5 shows the values of the constructive parameter of the CubeSat nanosatellite with different values of the maximum moment of inertia J_y depending on the static stability margin Δz.

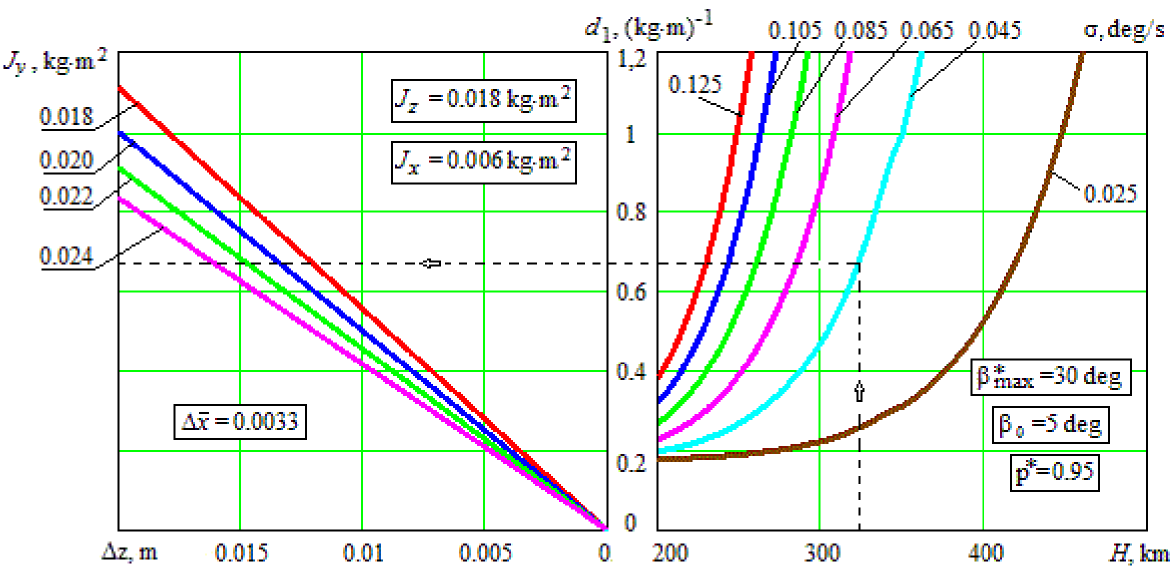

Fig. 5. The nomogram for choosing the constructive parameter of the nanosatellite with the gravitational-aerodynamic stabilization system

VI. Conclusions

The study presented in this paper was conducted in a probabilistic formulation of the motion dynamics for passive stabilization systems of different types. The types of classification of passive stabilization systems for CubeSat nanosatellites flying in a circular orbit have been proposed: aerodynamic, aerodynamic-gravitational, gravitational, and gravitational-aerodynamic systems. Each type corresponds to the altitude range of the dominance of a certain type of external moments and a type of stabilization (one-axis and three-axis). Analytical distribution functions of the maximum angle of deviation of the nanosatellite longitudinal axis from the required direction (orbital velocity vector or local vertical) have been obtained for uniform distribution and Rayleigh distribution of the components of the initial angular velocity vector. Based on the obtained analytical functions, the formulas were derived to select design parameters (geometrical dimensions, static stability margin, inertia moments), which ensure that the deviation of the nanosatellite longitudinal axis from the required direction is less than the allowable value, with a given probability at a required altitude for given spreads in the angular velocity caused by the separation system. Nomograms that allow selecting the main design parameters of the CubeSat nanosatellite, providing the required stabilization in low circular orbits have been constructed. The validity of the proposed solutions has been verified by using the spatial model for simulation of nanosatellite motion.

The obtained results provide practical guidance for developers of 2U and 3U CubeSats, which makes it possible to ensure the required orientation of the nanosatellite in order to minimize energy costs for keeping the orientation after quenching the acquired initial kinetic moment after separation by selecting the design parameters.

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